A "Lincoln Hat remap" in Illinois is even closer to happening. Because of upcoming Census Bureau reporting deadlines and state deadlines for redistricting, Illinois Democrats and Republicans might each have a 50 percent chance of drawing legislative lines in 2021 that will remain in effect for the next ten years.

So, how did we get here?

Just this week the Census Bureau announced additional delays for two upcoming target dates. First, the bureau said it hopes to deliver total state population counts by April 30. This number determines how many seats each state will get in Congress, but cannot be used on its own to draw congressional or state legislative seats. The more detailed demographic data used for redistricting will arrive even later. It's expected to be delivered no earlier than July 30.

This timeline is a potential problem for the state of Illinois.

Members of the Illinois General Assembly will not have detailed population data to draw new maps before June 30, a deadline set in place by the state constitution for having maps approved. Without new data, legislators will not be able to draw new maps that account for population shifts or ensure compliance with the Voting Rights Act. Without maps approved by June 30, the constitution sets in motion a backup plan -- an appointed commission of four Republicans and four Democrats will take charge of the remap. Those people are appointed by legislative leaders.

Historically, this appointed commission has not succeeded: three out of the four times when a backup commission was used, commissioners were deadlocked and needed a tiebreaker. Under the tiebreaker process, one new prospective commissioner from each political party is nominated and one of those two is randomly chosen and added to the commission. Therefore, if the Republican's name is drawn, Republicans would control the map drawing, even though Democrats hold supermajorities in the Legislature. This random drawing also extends the process into the fall, coming close to filing deadlines for upcoming elections in 2022. Potential candidates need to know what districts look like to determine whether they want to run.

Clearly, we could be in for a messy redistricting process this year, and we can all agree that the 2021 remap needs to be more transparent and people-focused than past redistricting cycles. Illinoisans need an opportunity to have a say in the final shape of their districts.

As the General Assembly sorts out its redistricting process, check out our We Draw the Lines effort with Representable, which gives you the opportunity to define your own community and better advocate for your community during the 2021 remap process.

Thank you for continued support, Madeleine Doubek


# Rethinking Redistricting 

A Discussion About the Future of Legislative Mapping in Illinois


The Institute of Government and Public Affairs (IGPA) is a public policy research organization based in all three University of Illinois campus cities. IGPA's mission is to improve public policy and government performance by: producing and distributing cutting-edge research and analysis, engaging the public in dialogue and education, providing practical assistance in decision making to government and policymakers. The institute's work not only advances knowledge, but also provides real solutions for the state's most difficult challenges. IGPA plays an important role in assisting government to better serve the public good. IGPA provides access to top-quality University of Illinois research to improve decision making at every level of government.

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# Preface: Posing Important Questions about Redistricting in Ilinois Robert F. Rich 



Once every 10 years, states engage in what is known as redistricting or drawing the electoral map. This process has great influence on or even determines the outcome of elections over the 10 years that follow. Redistricting is a task necessary to reflect the changes in population of a state over the previous 10 years as documented by the United States Census. Even though redistricting is so important, the general public generally seems to have little interest in or knowledge of how this process works. Politicians, on the other hand, understandably consider this process to be an incredibly important. Commentators and scholars have often labeled the mapping process as "gerrymandering" designed to draw districts which ensure the success of one political party at the expense of the other. The evidence for gerrymandering is odd-shaped electoral districts which are allegedly designed to insure for a specific political outcome. Hence, charges of unfairness and drawing of non-competitive districts are often levied. One very important and controversial question is: what constitutes a fair map? It is unclear whether the redistricting process can ever be impartial. The goal of this report is not to identify a single standard of fairness.

In this context, the Institute of Government and Public Affairs (IGPA) initiated a major project focusing on the redistricting process. The IGPA project is designed to be educational. On the one hand, we wish to provide background information about how the redistricting process is working in all states and about public preferences and knowledge with respect to redistricting. On the other hand, we engaged in the process of drawing maps based on a fixed-set of criteria. We implicitly posed the question: as we compare our maps to the ones adopted in 2000 and 2010, what are the major differences?

Our redistricting project examines redistricting from several different perspectives: a) a public opinion survey designed to examine the public's knowledge of redistricting and preferences for how this process should work; (b) drawing a computer-generated map of Illinois General Assembly districts based on three major criteria: compactness, contiguousness, and compliance with the 1965 Voter Rights Act; (c) comparison of the computer-model map with the actual 2000 map and with proposed 2010 maps; (d) drawing alternative congressional district maps and comparing these to what was adopted in 2000; and (e) analyzing "best practices" from all 50 states.

## This report is organized into five chapters:

1. The first chapter written by Professor Brian Gaines examines what is meant by fair redistricting? Gaines focuses on several different criteria which might be employed to determine fairness: equal population across districts, contiguousness, competiveness, continuity or deviation from the status quo, uniting people with common interests (based on economic, demographic, or geographic grounds), incumbency, proportionality of seats, and ensuring ethnic representation. The author underscores a key point: "with these criteria in mind, the fairness of a set of electoral districts can be assessed in myriad ways. There is a question of the object: to whom should maps be fair?"
2. The second chapter written by Professors James Kuklinski and Brian Gaines focuses on survey of 500 registered Illinois voters undertaken in May of 2010. The survey results convey a complex picture of the electorate and its preferences and knowledge of the redistricting process. The authors conclude
that Illinois registered voters clearly "do hold beliefs and preferences related to redistricting. They overwhelmingly favor an independent, nonpartisan commission whose members do not directly participate in politics to draw new district maps. ... Democrats and Republicans alike, rather than supporting redistricting plans that favor their own parties, prefer those that are not engineered to achieve particular political results."
3. Professor Hayri Önal and graduate student Kevin Patrick have contributed the third chapter which presents districting maps obtained from a computer model. A mathematical optimization model is used to configure 59 state senate and 118 House districts. The authors then compare the actual 2000 map with their map. They also propose a map for 2010 based on four major criteria: population equality, compactness, contiguousness, and compliance with the Voter Rights Act of 1965 . The authors point out that their computer-generated models are not designed to replace human decision-making but, instead, are designed to facilitate it.
4. Professors Nathan Anderson and Robert Kaestner, along with graduate student Hanqing Qiu, present a "method of identifying and measuring the extent of gerrymandering at the Congressional district level." They generate a "sample of nonpartisan, randomly drawn congressional maps to generate an 'electoral map' distribution of socioeconomic and demographic characteristics." They use the 2001 congressional redistricting process as the empirical example presented in this chapter. The authors reach a very interesting and important conclusion: "gerrymandering is widely viewed to exist and evidence to support that belief often consists of illustrations with odd-shaped electoral districts and references to the large number of safe seats. However, oddshaped electoral districts and safe seats may result naturally from the geographic distribution of voters combined with the legal requirements of drawing electoral districts..."
5. Professor Christopher Mooney is the author of the final chapter of this report. The chapter was originally published as part of IGPA's publication The Illinois Report 2011, and provides an overview of redistricting processes in the 50 states and to identify some "best practices." Mooney concludes that by taking a comparative state view, "we see that redistricting here (in Illinois) has been no more controversial than in other states, especially large and complex states."

## In this report we expect several questions to be addressed:

1) Can the Illinois redistricting process be improved?
2) Can the redistricting process be fair to all?
3) Is there a best practice from another state(s) that Illinois should adopt to make its redistricting process better?
4) Do computer models produce fair and proportionally represented maps?
5) Can computer models be used to complement the human element in redistricting decisions?

We at IGPA began this process more than a year ago knowing that it was not our intention to change the politics of redistricting. But at the heart of our effort was a desire to better inform those who participate in the political process about the exercise of democracy, and to better inform the voters for whom redistricting is intended to protect. The process and the debate will go on but we believe that our work can help improve redistricting in years ahead.

It also is clear to us that there is no one correct answer to what constitutes a "fair" redistricting process. The answer to this question depends on what criteria are being used and who is replying to the question. Moreover, we could not point to a best practice from another state that Illinois should definitely consider or adopt.

Our experience also suggests that the process we used to design a computer-generated map is one that can complement human decision-making and should be considered for redistricting in future years.


The year immediately following each decennial census looms large in American politics, as officials in all 50 states redraw electoral districts for their 99 legislative chambers and, in the 43 states whose populations entitle them to elect more than one member to the House of Representatives, revise the boundaries for U.S. House districts as well. For those who participate in or closely follow politics, the every-decade redrawing of legislative districts is one of the most important activities undertaken in American politics. They understand that redistricting will affect the electoral fortunes of the two major parties throughout the next decade, and thus that it will strongly shape the policies that Congress and the state legislatures pass into law during that time. They know that its effects will pervade political debates, over jobs, taxes, education, health care, collective security, and individual rights and liberties. To paraphrase a prominent scholar of the past, redistricting sets the framework for deciding "who gets what, when, and how."

Given this importance, it is perhaps surprising and definitely unfortunate that the very politicians who will lose or benefit from redistricting make the main decisions in most states. It is also noteworthy that, despite the crucial importance of districts, there is no firm consensus on how best to evaluate the rightness of a redistricting plan. Intuitively, we expect a "fair" drawing of districts, which implies that a plan should not favor one party over another, treat social groups differently, or be crafted to help or hurt any particular incumbent legislators. When public discontent is high and widespread, elections should be sensitive to this sentiment. Indeed, the 2006 and 2010 congressional elections suggest that voters were able to hold members of Congress accountable. The two elections delivered, respectively, a "thumping" of the Republicans and a "shellacking" of the Democrats, in the words of the two presidents humbled by the results. Yet
the U.S. House districts were not redrawn in between 2006 and 2010, so clearly the existing set of districts across the country did not preclude big swings in seat shares for the two major parties. Of course, since redistricting is done state by state, the U.S. House map is the handiwork of a great many individuals, varying in whatever political motivations they brought to the task. State legislative maps might thus be a better place to see fairness, or its absence.

In that light, consider Figure 1, which compares the 1982-90 and the 1992-2000 maps for the Illinois Senate and House. On the left are two sets of 59 Senate districts; on the right, two sets of 118 House districts. In the figure, these districts are not matched by their arbitrary numbers or by geography, but by partisanship. That is, for each map they are sorted by the proportion of the 1990 gubernatorial vote won by Jim Edgar. In that election, Republican Edgar narrowly triumphed over Democrat Neil Hartigan, 51 percent-48 percent. Those overall percentages are not, of course, reproduced in geographic units like counties, because there is a "natural" clustering of political preferences insofar as factors such as whether one resides in a rural, suburban, or urban locale correlate with vote choice. Electoral districts, unlike counties, are revised, so one can compare the two sets, before and after redistricting, to see how and where this revision had political consequences. Since the measures of Republican-ness used here is votes in a single fixed election, any deviation from the 45-degree line represents a difference in the two distributions of Republican vote that is due entirely to choices made by the map-makers about where to place boundaries.

The 1980s map was drawn by a commission of five Democrats and four Republicans, and was widely regarded as a Democratic gerrymander. The 1990s map was drawn by a commission of five Republicans and four Democrats, and was
widely regarded as a Republican gerrymander. ${ }^{1}$ In that light, it is not surprising to see in the figure that the 1991 boundary revisions increased the number of seats that had more Edgar votes than Hartigan votes from 29 to 31 in the Senate and from 56 to 67 in the House. Moreover, the "bumps" in the vicinity of the 0.5 region represent a substantial increase in Republican leanings in the marginal seats typically at play in a normal election. One might expect Republican map makers to choose lines that aim to transform expected ties or expected narrow losses into expected narrow wins, and this is what they seem to have done. ${ }^{2}$

It is quite easy to conclude that the 1990s map was friendlier to the Republicans, while the 1980s map was better for Democrats, in the aggregate. But which map was more fair? That question is much harder to answer, even if we limit the definition of fairness to cover only fairness to big parties. The ideal way to quantify fairness might be to compare each map to the entire set of possible maps, in terms of all relevant criteria. The 1990 Edgar vote is merely a convenient proxy for normal Republican vote, and should not be confused with actual Republican vote share in subsequent legislative contests. Staying with it, just the same, a natural question is whether there were other ways to divide the state into 59 Senate districts, each of which comprised two House districts, that would have boosted the total number of seats with Republican majorities even more? The answer is almost certainly yes, and there were also surely alternative maps that would have lowered the number of seats won by Edgar. However, it turns out to be impossible to devise a method for constructing the entire distribution of number-of-majorityEdgar seats, in order to observe the minimum and maximum possible values, and thereby say precisely how unusual, fair or unfair are the values 29 and 31, 56 and 67 . The problem of characterizing the full population of possible maps on any given criteria is NP-complete, or computationally intractable. Any given small example might be solvable, but the general problem is too hard to be solved by brute force, not matter how sophisticated and fast a computer is available. ${ }^{3}$

Faced with mathematical proof that the redistricting

[^0]problem is, in this specialized, technical sense, "unsolvable," one might conclude that anything goes, and fairness is thus an impossible or at least impractical standard for maps. That conclusion would be a severe over-reaction. Even without being able to produce a set of figures showing how a particular map compares to all of its potential rivals on various relevant criteria, one can subject maps to careful, albeit not perfect, scrutiny.

It is thus useful to review what criteria might be applied in assessing the quality (fairness) of an electoral map. A brief summary of the traits most often raised and their current legal status follows.

Census population equality. American courts began policing malapportionment in the mid-1960s, following a series of landmark cases including Bakerv Carr and Reynolds $v$ Simms. At that time, the major tension surrounding districts was the infrequency of boundary revision, and the consequent over-representation of rural areas and underrepresentation of urban areas. Courts now require a very high degree of uniformity in district population at the time of map construction, based on the prior census. As time passes, district-to-district discrepancies grow, sometimes very dramatically, but courts have so far insisted only on temporary conformity to a representation-by-population standard, ignoring the fact that none of the five elections in a normal decennial apportionment cycle will feature equally populous districts, given that the population data used by the map makers is already two years old by the time of the first election on the new map. Exactly how much discrepancy from equal population across districts is tolerated varies by jurisdiction, but approximate equality is a strong constraint for all redistricting. ${ }^{4}$

Contiguity. An even more binding constraint than equal populations is that districts must consist only of contiguous territory. No American jurisdiction permits districts whose component areas are not connected. This constraint is merely a technical point, and not an open political matter, with the exception of some very minor, technical issues pertaining to such odd situations as when islands must be connected to mainland territory in a single district.
${ }^{4}$ An interesting question not plainly resolved at present is whether maps can, in some indirect manner, take into account projections of future population growth. A different, potentially huge issue is whether any court will ever conclude that fair apportionment requires not equal populations but rather equal populations of eligible voters. The oft-repeated catchphrase "one person, one vote" (from Justice Douglas's majority opinion in Gray v Sanders) is, after all, a statement about voting power not actually met as long as districts are aligned in population rather than potential voters.

Figure 1: Comparing Two Illinois General Assembly Maps


Compactness. Individual districts can be evaluated for the simplicity, or compactness, of their shape, and it is routine to characterize maps in terms of some overall aggregation of district compactness. Oddly shaped districts do not violate contiguity, but they usually involve linking otherwise noncontiguous regions with narrow, nearly empty corridors. Where contiguity is thus just barely met, as when districts include segments running along rivers, highways, or unpopulated areas like golf courses, the resulting districts (and, in turn, map) are normally low in compactness. Many states require that districts be compact, but do not specify how to measure compactness. There is a very technical literature on how best to measure the compactness or simplicity of maps, and there is no consensus on a single best measure. Compactness is potentially in tension with most other criteria, particularly the norm that existing boundaries of counties, cities, townships, precincts, and the like, ought to be respected as much as is practical.

Competiveness. Arguably the most important trait of a district is whether it features competitive election contests. At the end of an apportionment period, one can look backwards and simply count how many races in each district were close, by some standard (e.g. margin of victory under 10 percentage points). To complicate matters, one might attempt to decompose election results into components and isolate the effect of the district "normal" vote from year-to-year tides, candidate effects, spending, and so on. Moreover, assessing the competitiveness of a district before elections have taken place requires some kind of forecast, as

from a statistical model extrapolating past election returns or predicting votes from electorate demographics. Another important distinction is whether competition is desirable at the level of the district, in the aggregate, or both. A map consisting entirely of safe seats can be finely balanced in terms of seat totals and control of the relevant legislative chamber, and these surely matter most in policy terms.

Continuity. From the point of view of fairness, the merit in privileging the status quo depends on the fairness of the prior map. But a variety of arguments can be advanced in favor of not shifting electoral lines too much, if possible. For instance, it is well known that most voters pay little attention to politics, and there might be advantages to stability in boundaries by way of greater awareness among voters of who represents them.

Communities of interests. Another fairly common principal in normative debates is that districts should be drawn to unite people with common interests. As a standard, this norm is clearly very elastic and depends on specifying what kind of interests matter. The "communities" in question might, for example, be defined on economic, demographic, or geographic grounds. Under this rubric, one can imagine a variety of claims. "This retirement community has nothing in common with that mining town, so don't put them in the same district." "These districts dilute rural power, privileging the city too much." "There's no reason to split up so many neighborhoods."

Current boundaries. Many argue that maps ought to respect pre-existing boundaries as much as possible. In other words, counties, cities, towns and the like should be split as little as manageable, and electoral district boundaries should coincide with these other boundaries whenever possible, given the contiguity and population constraints. The rationale could be merely logistical, to facilitate matching different kinds of data; however, the goal of using existing lines is more often described in terms of communities of interest or continuity. Or, again, this principal can be understood as embracing a constraint on map makers simply to limit the possibilities for politically motivated selections of lines.

Incumbency. It is difficult if not impossible to defend protecting current office-holders as a normative principal of map making, except perhaps insofar as doing so might promote some other values, such as continuity. Nonetheless, incumbent protection is still commonly asserted to be a prime motivator in a large share of all U.S. electoral maps. Note that partisan gerrymandering is quite distinct from incumbent protection, and at least partially in tension with it. A party bent on increasing its own seat share will make a minority of seats safe for its opponent, and might, thereby favor the disfavored party's incumbents. But the goal is not to help incumbents as such, but to make the other party's vote inefficient, i.e. to ensure that they "waste" a lot of votes in needlessly large margins.

Proportionality of seats. An intuitive, or perhaps naive, sense of fairness is that parties ought to win about the same proportion of seats as they win votes. Whatever the benefits of competition, few would argue that a state in which party A wins 70 percent of the vote and party $B$ wins 30 percent of the vote ought to see very close seat totals for A and B. It is clear, however, that plurality elections (those in which the candidates with the most votes win) can, and usually do, inflate the vote shares of stronger parties into larger seat shares. With the simplicity of two-party politics, the U.S. normally sees one winner and one loser in the translation of votes into seats. A follow-up question is whether a given map treats parties alike. If they split the vote evenly in some election, do they split the seats evenly (at least in expectation)? In successive elections, does party A win 75 percent of the seats from 60 percent of the vote while $B$ wins only 65 percent of the seats when it secures 60 percent of the vote? Political scientists tend to focus on such discrepancies in disproportionality, or "bias," as a key measure of a map's fairness.

Proportional representation. A different kind of proportionality has lately been extremely important in the politics of redistricting in the U.S. Although the jurisprudence is somewhat fuzzy, it is clear that courts now tolerate, and sometimes demand, that districts be constructed so that
racial groups- blacks, Hispanics, and possibly now Asiansconstitute majorities in many districts. Ongoing controversy surrounds the question "How many?" The maximum number possible? Sufficiently many that it is highly likely that the proportion of the legislative chamber that is black/ Hispanic/Asian is no less than the share of the district population? Deliberate racial gerrymandering, though it has become understood as required by the Voting Rights Act, sits awkwardly with any argument that districts should be drawn on a non-political basis. There can be direct partisan implications, as when the creation of very safe majority black Democratic seats (expected to return black members) reduces the total number of seats that can be won by Democrats. Feasibility depends not only on population shares, but on the degree of segregation as well. The current Illinois 4th U.S. House district demonstrates how construction of a majorityHispanic seat in Chicago was possible, but only by doing great violence to compactness. Residency patterns can make it difficult to avoid choosing between rival majority-minority seats, e.g. "either one extra black seat or one extra Hispanic seat can be drawn, but not both."

With these criteria in mind, the fairness of a set of electoral districts can be assessed in myriad ways. There is the question of the object: to whom should maps be fair? To (major/all) parties, candidates, incumbents, voters, subsets of voters (as defined by locale of residence, race, voting inclinations,....)? Or, perhaps as many of the preceding as possible? The brief discussion of the districts for the Illinois General Assembly above introduces both outcomes (election returns at the district level) and process (who drew the maps), and both are frequently broached as fair or unfair. Without attempting an exhaustive categorization, the notions of fairness that seem to dominate policy discussion include: non-partisan process; bipartisan process; apolitical process; proportionality between parties' seat and vote shares; high degrees of competitiveness, district by district or in the aggregate; and fairness in expected representation of racial groups.

Interestingly, one debate playing out among scholars, expert witnesses and judges in court proceedings, and probably within legislative chambers, is whether fairness ultimately requires apolitical districting or, rather, political districting openly aimed at some explicitly political measure of what it means to be fair. In the former view, maps should be drawn blind to political considerations such as where incumbents live and how past elections turned out in each sub-region. A key feature of the process should be a prohibition against political data being used in any way by map makers. The latter view, in contrast, holds that truly fair maps must be engineered to be fair, and that feigning ignorance is pointless. The best way to achieve competitiveness (or proportionality, etc.) is to produce the best possible estimates of the expected competitiveness (or proportionality, etc.) of rival maps,
and then to select whichever map maximizes this (these) attribute(s), subject to the technical limitations mentioned above.

In turn, common rebuttals run something like the following. To those who insist on openly engineering expected outcomes, politically blind map making is never neutral, but, at best, haphazard in its unfairness, and, at worst, merely cagey, insofar as experts can make very good guesses about voting patterns even without formally using the data. Advocates of depoliticizing the process can counter that the appearance of finely tuned fairness in any engineered map is misleading, and relies on experts to produce complicated
estimates of normal votes, expected turnout, and other difficult-to-measure traits. Behind the smoke and mirrors will typically be a partisan scheme masquerading as technocratic impartiality.

As this debate over how to balance process and expected outcomes continues, it is perhaps simplest to conclude that one of the surest signs of an unfair map is one created in secret. No one defends lack of transparency as somehow being fair or justifiable.


# What Does the Public Know About Redistricting? What Does the Public Want from Redistricting? 

Brian J. Gaines and James H. Kuklinski


Given that redistricting strongly shapes the politics and policymaking in the 10 years that follow, one might expect citizens to pay heightened attention to it. Conventional wisdom says quite the opposite: most citizens view the redrawing of electoral districts as an arcane, technical task that does not warrant their attention; and, therefore, they express no interest and give no attention at all. Scholars of public opinion take this ignorance or indifference on the part of the public for granted, and so almost never raise the topic with survey respondents. In the gigantic literature on how electoral boundaries are drawn and should be drawn, there is hardly any reference to public opinion. Here, we demonstrate that leaving the public out of the discussion is not necessary. In May 2010, the Institute of Government and Public Affairs conducted a survey of 500 registered Illinois voters. ${ }^{1}$ At the time, redistricting was emerging as a salient news topic; the League of Women Voters, the Illinois Chamber of Commerce, the Illinois Farm Bureau, and others were promoting a petition to place a constitutional amendment changing the redistricting process in Illinois onto the November 2010 ballot. Despite considerable media attention, the petition drive for their "Fair Map Amendment" did not collect enough signatures to place the item on the ballot.

The survey results, some of them reported below, convey a complicated picture. Ordinary citizens are, indeed, fairly ignorant about how electoral maps are made. Their lack of knowledge, however, does not indicate a belief that districts

[^1]do not matter. More importantly, there are reasons to believe that ordinary people have preferences for how districts should be drawn. In that regard, they are interested in the process, even if details on the process rarely filter through to general awareness. Even more interesting is the evidence that the public lacks inclination to exploit maps for partisan gain. For example, even partisans prefer simple maps, and do not like the idea of parties using complicated districts to disfavor their opponents and thereby gain long-term electoral advantage.

## Ignorance

The survey asked respondents if they knew how both the U.S. House districts and the General Assembly districts that were used for elections held between 2002 and 2010 were drawn. For both questions, respondents were given a list of possible answers, including, of course, the correct answers. Yet in both cases, 82 percent said that they did not know. Of the remainder, about half (roughly 10 percent of respondents) knew that the U.S. House map was drawn by a bipartisan team of incumbents and only a handful (about 3 percent) knew that the General Assembly map was produced by a commission with a majority of Democratic members. We asked this question to probe the depth of people's knowledge, although, in fairness, we did not expect high percentages to know the right answer to a question about a not-well-publicized procedure that takes place only once a decade, and had occurred almost 10 years prior to the survey. This said, the result validates the view that redistricting is too specialized a topic to attract much attention even of registered voters.

The survey also asked respondents a series of questions about the Fair Map Amendment that a coalition was, at the
time of the survey, trying to place on the November ballot. About three-quarters of our respondents said they had not heard about the petition drive. The effort received only modest media coverage, so, again, this lack of awareness was not terribly surprising. Still, responses to these two questions convey an image of an electorate almost completely disengaged from one of the most important processes in Illinois politics. However, this image fails to capture the whole truth.

## Impact and Importance

One might reasonably assume that citizens don't try to learn about redistricting because, unlike those close to politics, they believe that redrawing districts is an inconsequential act. That is, ignorance could stem from indifference. This is definitely not the case. When asked how much the drawing of new legislative districts affects who wins and who loses in subsequent elections, about 40 percent said it has a very big impact, and nearly 40 percent more said it has some impact.

Our survey, then, uncovered an electorate that thinks districts matter, but whose members do not know how they get drawn. Are they apathetic because they think the process is too difficult to understand, and something about which ordinary people cannot hope to have informed preferences? No.

## Interest and Preferences

Although most Illinois registered voters seem to know little about the specifics of redistricting in Illinois, a large majority hold beliefs and preferences directly relevant to it. Although few knew about the Fair Map Amendment, for example, sizeable majorities supported the specific provisions of it when told what those provisions were. More than 60 percent, for example, supported the provision to draw House and Senate maps independently, rather than "nesting" two House districts in each Senate district, as at present. An equal percentage endorsed the provision to create a nine-member commission to redraw the district maps, including four commissioners who are not lobbyists or political officials.

In the same vein, respondents were asked who they thought "should be responsible for drawing the electoral districts for the state legislature." Nearly 50 percent chose "an independent, nonpartisan commission whose members do not directly participate in politics," and another 17 percent chose "a computer program prepared by someone outside of politics." Less than 3 percent chose "the state legislature and the governor," and less than 5 percent chose "House districts should be drawn by the Illinois House and Senate districts by the Illinois Senate." Eight percent favored "a bipartisan commission consisting of equal numbers of Democrats and

Republicans." Very simply, a large proportion of Illinois registered voters prefer a redistricting process that does not allow partisan politicians to participate in it, which is to say they prefer a process that differs dramatically from the current one.

Registered voters thus have preferences about the redistricting process, and they prefer that it not be partisan. In the survey, we also explored their preferences for outcomes by having them rank a variety of possible criteria for good maps. The specific wording was as follows.

There are many possible ways to think about what constitutes a good set of electoral districts. Please rank the following goals:
A.Districts should take relatively simple shapes.
B. Boundaries should follow existing country and city lines as much as possible.
C. As many districts as possible should be about equally balanced between Democrat and Republican voters.
D. The percentage of seats won by each major party should be about the same as its percentage of the total vote.
E. Towns and neighborhoods that have a lot in common should be put in the same district as much as possible.
F. The proportion of legislators that is black should be about the same as the proportion of the population that is black.
G. The proportion of legislators that is Hispanic should be about the same as the proportion of the population that is Hispanic.

In the common jargon surrounding redistricting literature, Option A is, roughly, the goal of "compactness." Options $B$ and $E$ are varieties of the "communities of interest" argument. Option C is the goal of "competiveness" at the district level, while Option D is seat-vote proportionality, a common (though contestable) baseline for fairness to parties. Options E and F describe racial gerrymandering, presently understood to be more or less required to some degree by the Voting Rights Act of 1965.

Figure 1 summarizes the responses. (In the actual survey, the criteria were ordered randomly. In the figure and in the text above, we order them by what proportion chose it as most important.) For each of the seven items, the histogram shows what percentage of all respondents chose each ranking, from first to seventh (last). The numbers at the top of each

Figure 1: Rankings (from 1st to 7th) of Seven Criteria for Good Maps


A


E


B


F

panel are the average ranking, and a lower number reflects respondents ranking the particular goal more highly.

Illinois registered voters seem to place particular importance on two goals, creating compact districts and taking account of existing county and city lines to the extent possible. Very small percentages rank either of these goals as least important. To put it another way, sizeable percentages of Illinois voters presumably would prefer the map presented in the following chapter by Önal and Patrick-a map explicitly designed to emphasize compactness and which was drawn without any use of data on voting patterns or incumbents' residencesover the current and about-to-be-adopted actual legislative maps.

Two other goals-balancing Democrat and Republican voters in as many districts as possible and ensuring that the


G

percentage of seats won by each party is close to the percentage of the total vote it garnered-received notable support. Both of these goals relate, obviously, to ensuring some variety of fairness to the major parties. "Gerrymandering" is a term with many meanings, but the two most common forms of manipulating district lines are protecting incumbents and exaggerating the strength of one party by "packing" supporters of the other party into a small number of highly safe seats. The goal is to ensure that the favored party's seat share is much larger than its vote share, so such "partisan gerrymandering" is diametrically opposed to criterion D.

On the other side, Illinois registered voters consistently ranked guaranteed representation of Hispanic and black citizens as least important. These two results imply that most residents do not enthusiastically support, and indeed might oppose, the creation of majority-minority districts, a matter
that leaders in Hispanic and black communities deem as a very high priority in the creation of new legislative districts. ${ }^{2}$

Thus far, we have evidence that registered voters are ignorant about redistricting, but acknowledge that the matter is important, and have enough interest in it to endorse particular processes and criteria for good maps. We included one other question designed to measure public preferences over rival criteria for what makes a map preferable. The challenge began as follows:

Finally, please consider a hypothetical redistricting problem. In the perfectly square state pictured below, there are 64 square counties. Each county contains 100 people. 32 counties have 100 Democrats (those squares are white) and 32 counties have 100 Republicans (those squares are gray). The new electoral map must consist of 8 districts, each containing 800 voters. Please rank the 6 possible maps shown below from best to worst, on whatever grounds you like.


This was, of course, a highly stylized version of redistricting, but our goal was to confront respondents with the fact that different criteria often clash. It tends to be true, for example, that the creation of majority-minority districts is at odds with designing competitive seats. Making compact districts will sometimes tend to help one party and harm the other, whereas a map engineered to be fair in the sense of delivering seat shares that match vote shares is unlikely to be compact. The maps our respondents saw are shown in Figure 2 on the following page, along with a table summarizing their key features.

In summary, map A is quite clearly non-compact, but is strictly fair in seat proportionality-it is the only map that translates a 50 percent- 50 percent tie in votes into an expected tie in seats. It is also high in district competitiveness. Map B

[^2]is clearly compact, slightly biased in favor of the Democrats, and low in competitiveness. Map C is debatably compactit scores poorly based on total boundary length, but would do better with more complicated measures of compactness, given its use of simple shapes. It is slightly biased in favor of Democrats, and high in competitiveness. Map D is clearly non-compact, very biased in favor of Republicans, and medium in competitiveness. Map E is compact, simple, slightly biased in favor of Republicans, and medium-high in competitiveness. Map F is clearly non-compact, very biased in favor of Democrats, medium-high in competitiveness.

Which map did respondents like? Overall, the average ranks were as follows: A 3.82 (4th best); B 2.90 (2nd best); C 3.19 (3rd best); D 4.10 (5th best); E 2.76 (best); F 4.14 (worst). Again, lower is better, since a score of 1 means the map was ranked best, and 6 means it was ranked last. The respondents embraced the simplest maps, consistent with their expressed preference for simple shapes as discussed above. While map A is uniquely fair to the Republican and Democratic parties, it was not very highly ranked, presumably because respondents do not like twisty shapes, regardless of their rationale. Surprisingly, even the most partisan of our respondents did not express a preference for maps that would help their party to win seats. There was very little difference in how selfidentified Republicans, Democrats, and independents rated the maps, with E and B always coming out on top. ${ }^{3}$ Finally, it seems that voters in Illinois are serious when they say that they do not like contrived maps. The benefit of simple shapes is that they are prima facie evidence of a lack of manipulation. Even strong partisans do not like to see politicians draw twisty districts in an effort to win at the maps, rather than at the polls.

## Concluding Comments

Public opinion has not been a factor in Illinois redistricting. In fact, no one has even bothered to measure it. Nearly everyone, it seems, assumes that ordinary citizens lack sufficient knowledge about the redistricting process to hold meaningful beliefs and preferences. Some of the IGPA survey results confirm this assumption. When it comes to knowing the specific process by which Illinois redraws its districts, a very large percentage of registered voters fail. Nor were many aware of a petition drive, in high gear at the time of the survey, to place on the ballot a constitutional amendment that would have changed the way electoral districts are drawn in Illinois.

Those who control the current redistricting process, namely the two Democratic legislative leaders and Governor Pat Quinn, might have dodged a bullet. The survey results reveal

[^3]Figure 2: Alternative Maps for Stylized Map-Selection Question


Table 1: Traits of Alternatives in Map-Selection Item

| Map | Compactness Rank | Simplicity | Seat Split | Competitive Seats |
| :---: | :---: | :---: | :---: | :---: |
| A | 5th (83) | Low | 1 R. - 1 D. - 6 ties | $6 / 8$ expected close |
| B | 1st (64) | High | 3 Rep. -5 Dem. | 3/8 expected close |
| C | 6th (88) | High | 3 Rep. -5 Dem. | 7/8 expected close |
| D | 4th (78) | Low | 2 Rep. -6 Dem. | $4 / 8$ expected close |
| E | 1st (64) | High | 5 Rep. -3 Dem. | $5 / 8$ expected close |
| F | 3rd (76) | Low | 6 Rep. -2 Dem. | $5 / 8$ expected close |

Notes: Here, we measure compactness by boundary length, which is the number in parentheses in column two. Lower scores indicate higher compactness levels. There are many alternative measures, more complicated to compute. Accordingly, the second column reports a qualitative assessment of the simplicity of the district shapes, to reflect the fact that map $C$ scores poorly on total boundary lengths, but does not have the contrived appearance of maps $A, D$, and F. Vote totals, by design, are 50 percent Republican and 50 percent Democratic, so read column four knowing that perfect proportionality between votes and seats requires a 4 Republican- 4 Democrat split, or, equivalently, any split with equal numbers of seats having marjorities of Democrats and Republicans plus ties in the remainder of the seats. The "ties" are seats containing four Republican and four Democratic counties. For column five, we define "close" as any district with expected vote of 500-300 or 400-400, and treat all other options (800-0; 700-100; 600-200) as not close/competitive.
that, had more people known about the petition drive, it might well have been placed on the fall ballot; and had it been placed on the ballot, it likely would have passed. When told about specific provisions of the petition, most respondents supported them. That amendment would not have altered the process of drawing maps to use from 2012 to 2020, of course. Because Democrats now control both chambers of the General Assembly and the governorship, they are in a position to gerrymander without having to win a lottery for which party gets to unbalance a previously balanced commission, as in 1991. What does the public want from its maps?

Illinois registered voters clearly do hold beliefs and preferences related to redistricting. They overwhelmingly favor an independent, non-partisan commission whose members do not directly participate in politics to draw new district maps. They think that legislative districts should be compact and that their boundaries should follow existing county and city lines. Democrats and Republicans alike, rather than supporting redistricting plans that favor their own parties, prefer those that are not engineered to achieve particular political results. In short, a sizeable majority of Illinois registered voters endorse the very things that those politicians in control of redistricting avoid, and reject the very things that these same politicians currently do, and have been doing for a long time.

Herein lies the cost of Illinois citizens' lack of attention to redistricting. One can only presume that voters throughout the state would be expressing criticism of the current redistricting process if they knew fully how it works. In addition, the lack of surveys and media reporting of the results makes it difficult for any one citizen to know that many others share his or her beliefs and preferences. It would be wrong to free citizens of all responsibility; they cannot hold officials accountable unless they pay attention. Perhaps even more fundamentally, the media need to play a more useful role. The complexity of the redistricting enterprise assists politicians who draw rigged, unfair maps while pretending to do otherwise and to be listening to what the public wants. But the process is not particle physics or number theoryit is not so complicated that the public could never hope to understand the tradeoffs and tensions, and the various ways to be fair or unfair about where the lines go. Sadly, it is too late for the public's views to be brought into the redistricting process this time. For another 10 years, the citizens of Illinois will channel their political choices through maps designed behind closed doors by partisans acting on partisan goals.


TThis chapter presents a systematic approach to legislative redistricting in Illinois and alternative districting plans obtained from a computer model developed by researchers at the University of Illinois and the Institute of Government and Public Affairs. A mathematical optimization model is used to configure 59 State Senate districts (SSD) in such a way that: i) all districts have almost equal populations (allowing up to 5 percent deviation from the average district population); ii) each district is spatially contiguous and compact; iii) communities are divided to the minimal extent so that a specified number of districts can be configured as majority-minority districts where at least 50 percent of the population is comprised by African-American or Hispanic residents. Each SSD is then divided into two House of Representative districts (HRD), again using the same criteria when configuring the HRD district boundaries.

The model assumes census tracts as indivisible spatial units (building blocks) and assigns every tract to a district in such a way that the total distance between all tracts in a district and a centrally located tract in that district is minimized while satisfying the districting criteria listed above.

A 'reasonably large' set of tracts is identified as potential district centers (seeds). The model chooses the best subset of those candidate centers and assigns tracts to them simultaneously while considering other districting criteria as well. This promotes compact district shapes to the extent possible around the selected centers. The only input used when identifying the seeds and during the tract-todistrict assignment process is the spatial distribution and demographic characteristics of the population, no political or group interests are taken into consideration. Therefore, the procedure is transparent and free of partisan biases.

The model is applied to the last two census data sets and alternative district maps are generated.

Because the political process of mapping the new state legislative districting maps was in progress at the time this report was written, the model-generated maps based on the 2010 census data could not be evaluated and compared against them. However, to demonstrate the merits of the systematic districting procedure described here, the maps generated by the model using the 2000 data are compared with the latest state legislative districting maps in terms of spatial compactness and minority representation. Two major findings are particularly noteworthy: i) the modelgenerated districts are considerably more compact than the current SSD and HRD plans; ii) the Hispanic population in the state was underrepresented in the current districting plan, which is significantly improved by the model generated districting plan. Specifically, only four out of 59 Senate districts had Hispanic majority, which corresponds to about half of the state's Hispanic population ratio ( 6.8 percent as opposed to 12.3 percent population ratio according to 2000 census data). The model could configure one more Hispanic majority-minority Senate district than the actual plan without compromising fundamental districting criteria and representation of the African-American minority group in the state. Likewise, the actual 2000 plan included eight Hispanic majority-minority House districts, whereas a proportional representation of the Hispanic population would require 14. The model could generate 10 HRDs with Hispanic majority, again without compromising other districting criteria including the representation of the African-American minority in the state.

## Introduction

Every 10 years following the release of census results by the federal government, district maps are redrawn by the states for legislative and congressional districts based on demographic changes affecting the spatial distribution of the population. While the number of congressional districts is determined by the federal government, based on the reapportionment process and 'one person-one vote' principle, the state legislators determine the district boundaries. On the other hand, both the number and boundaries of the state Senate and House of Representatives districts are determined by individual states according to the principles laid out in the state constitutions. This chapter focuses on legislative redistricting in Illinois and presents the results of a modeling study conducted by the researchers at the University of Illinois at Urbana-Champaign in collaboration with the faculty at the Institute of Government and Public Affairs. A novel computer-modeling approach is used to draw the district boundaries considering the fundamental districting criteria stated by the state Constitution and have been used in previous districting practices.

Determination of the district boundaries involves various criteria which vary from state to state. In Illinois, spatial contiguity and population equity are two essential provisions when drawing both legislative and congressional district boundaries. Additionally, Illinois districting rules require that the districts must be spatially 'compact,' although a precise and universal definition of the concept has not been provided. Another important consideration is minority representation, namely some districts are required to have a majority minority, which is typically around 60 percent. The number of such districts should reflect, to the extent practicable, the proportion of the respective minority population in the state's total population. Because district boundaries can have a substantial impact on election results, the redistricting procedure has always been subject to partisan politics and disputes have arisen particularly when the district boundaries are significantly 'gerrymandered' in favor of the legislative majority who prepared the plan. ${ }^{1}$

In some cases, the process of redistricting ended at the state Supreme Court which approved or invalidated a districting plan. It has been argued extensively in the political districting literature that an objective, systematic, automated process based on a minimal set of generally agreed principles would eliminate subjectivity and alleviate the role of partisan biases, facilitate the process of reviewing and evaluating alternative district plans, and ensure fair political representation (Weaver and Hess, 1963; Browdy, 1990). These arguments motivated

[^4]the study presented here. A computer-aided districting procedure involving a mathematical programming model (specifically linear integer programming) has been developed for this purpose and implemented using the census data and demographic characteristics of the Illinois population to draw the legislative district boundaries. Here we present the model-generated maps using the 2000 data and compare them with the existing districting plan to explain the merits of the systematic districting approach used in the study. The same approach is used to generate district maps using the 2010 census data, which are also presented.

## Systematic Political Districting

Systematic generation of political district plans by simultaneous consideration of spatial, social and demographic criteria is a challenging combinatorial optimization problem. Before the introduction of computers, district plans were drawn manually by state legislators using map making skills. Starting in 1960's, various algorithmic procedures and computer programs have been developed, and some implemented, to facilitate political redistricting in a systematic way (e.g. Vickrey, 1961; Harris, 1963; Weaver and Hess, 1963; Forrest, 1964; Hess et al., 1965; Nagel, 1965; Liittschwager, 1973). ${ }^{2}$ Although the earlier districting studies in 1960's and 1970's considered some sort of objective function, such as maximizing compactness, minimizing the deviation from an existing plan, etc., they employed rulebased heuristic algorithms instead of formal optimization to generate district boundaries. This was mainly because of computational convenience and ease of incorporating complicated districting criteria in heuristic procedures. While exact optimality is not guaranteed, heuristic approaches have been appealing because 'good' solutions can be obtained in little processing time depending on the quality of the algorithm employed and/or data characteristics. Some empirical studies have shown that the computer-generated district maps developed by use of heuristic procedures were substantially better than the existing maps in terms of either improved population equality or compactness, or both (Hess et al., 1965; Nagel, 1965; Garfinkle and Nemhauser, 1970; Morrill, 1981; Mehrotra et al., 1998). ${ }^{3}$ The progress made in computer hardware and software technology, particularly in the past two decades, now allows us to formulate and solve the districting problem by use of formal optimization

[^5]approaches to improve the performance of computer models further. This study presents a major step in that direction.

The computer model we developed and used here is a linear integer programming model. It assigns approximately 3,000 spatial units to 59 State Senate districts (SSD) ${ }^{4}$ employing the districting criteria described above, namely population equality, contiguity, compactness and minority representation. Consistent with the Illinois districting rules, each SSD is then divided into two House of Representative districts (HRD), thus forming 118 HRD districts, again using the same criteria when configuring the HRD district boundaries. We briefly describe here the modeling methods in a non-mathematical form. A formal mathematical representation of the model is provided in the Appendix for readers who are interested in the methodological and algebraic details.

The population equality requirement does not imply an exact equality of the district populations. Consistent with the previous districting practices and the state districting rules, we allow slight deviations, up to 5 percent, between the populations of individual districts and the ideal (state average) district population. Geographical contiguity is not a federally mandated requirement, but most state constitutions, including Illinois, require contiguous districts (Grofman, 1985; Levitt, 2008). Contiguity means that any pair of spatial units in a district can be connected through a path formed by mutually adjacent units. In other words, one should be able to travel within a given district without having to cross another district's boundary. Compactness is considered as a desirable quality in a districting plan, being a direct way to prevent 'gerrymandering' (Morrill, 1987; Grofman, 1985). A well-defined and universally applicable measure of compactness has not been given in the spatial analysis literature, neither does the state constitution provide a definition. Here we measure compactness as the sum of population-weighted distances from all spatial units in a district to a centrally located unit in the same district. The smaller the total distance the more compact the district. The overall compactness of a district plan is maximized by minimizing the sum of those total distances across all districts. In this respect, our modeling approach builds upon the moment of inertia method introduced first by Weaver and Hess (1963) and Hess et al. (1965), which is used later in several other districting studies (e.g., Morrill, 1981; Plane, 1982; Mehrotra et al., 1998; Garfinkel and Nemhauser, 1970).

This study differs from previous districting studies in several aspects. First, we use an exact (formal) optimization procedure

[^6]when configuring districts. Second, our model selects the district centers endogenously, that is, instead of fixing them up front, the model selects them from a reasonably large set of candidate central units. Selection of the district centers and the assignment of individual units to the selected centers are done simultaneously. Third, we configure a specified number of majority-minority districts for both African-American and Hispanic groups using the same districting criteria applied to regular districts; the majority-minority districts are required to have more than 50 percent of their population comprised by the respective minority population. Finally, we employ an explicit mechanism that guarantees spatial contiguity. The methodological details and brief explanations of the model equations are provided in Appendix.

## Data

We consider census tracts as the base spatial units (building blocks) and assume that no tract can be divided into smaller spatial units. Thus, a given tract is either entirely included in a given district or it is not part of that district at all. For spatial and demographic information about census tracts we use the Gazetteer files from United States Census Bureau data, including locations, populations and racial breakdown of the populations, and county associations. In Illinois, there were 2,964 census tracts in 2000. In 2010, this number was increased to 3,121 . Though most tracts are near the average population (4,190 in 2000 and 4,110 in 2010), there were some extremes. For instance in the 2000 data, 13 tracts contained 25 or fewer people while the largest tract had a population of 34,055 . We created an adjacency matrix and edge-to-edge distances between any pair of tracts, which are used in the model to ensure connectivity.

## Model-Generated District Maps

Several legislative district maps were obtained from the model considering the four requirements mentioned above. To demonstrate the merits of systematic redistricting, we first present the district plans (maps) produced by the model using the 2000 census data and compare them with the actual districting plan that was made using the same data set in terms of compactness and minority representation. We use the same metric when comparing the compactness of modelgenerated districts, namely the total population-weighted distance between all tracts in a district and a centrally located tract (centroid) in that district. For this, we determined a centrally located tract in each of the actual districts and computed the total population-weighted distance from all tracts in that district to the respective central tract.

## Districting with the 2000 Data

Using the 2000 census data, the optimal districting plan

Figure 1a: The model-generated Illinois State Senate District Plan. The solid black lines indicate the Senate district boundaries, red lines indicate the House district boundaries.


Figure 1b: Actual Illinois State Senate District Plan

obtained from the Model for Illinois Senate districting is displayed in Figure 1a for 'downstate' and Figure 2a for the greater Chicago area, respectively. For comparison, we also display the corresponding actual (currently implemented) district maps side by side in Figures 1b and 2b, respectively. As the two figures clearly demonstrate, the model-generated districts have much simpler shapes (less gerrymandering) than the actual districts, which is an indication of improved compactness. Based on this metric, we find that the modelgenerated map is about 15 percent more compact than the current districting plan.

The Voting Rights Act of 1965 put an end to many of the discriminatory practices that were used to keep minorities from voting. Preventing a person from voting based on literacy tests was outlawed and any states that had used these practices in the past or had less than 50 percent of the population registered to vote would need clearance from the Department of Justice before making any changes that would affect voting. The goal was to give minorities an equal opportunity to vote and, in turn, elect their preferred candidate. The Voting Rights Act manifested itself in
redistricting primarily by forcing states to use districting plans that contain (approximately) the same percentage of districts that are a majority of a minority group as the proportion of the minority group in the state population. This has been an important disricting criterion in Illinois when generating both state Senate and House districts. In the 2000 districting plan, eight of the 59 districts, roughly 14 percent, were comprised of greater than 50 percent black population. This reflected the proportion of the AfricanAmerican minority in the state, which was 14.4 percent in 2000. Likewise, four out of the 59 districts, roughly 7 percent, are a majority Hispanic. According to the 2000 census, 12.3 percent of the state population was Hispanic. Therefore, while representing the black minority fairly, representation of the Hispanic minority in the Senate was quite below the population share of the latter group. On the House side, the actual plan had 18 black majority HRDs, which corresponds to a slight overrepresentation of the black minority (15.3 percent of the total number of HRDs) while the number of Hispanic majority-minority districts was eight. The latter indicates again an under-representation of the Hispanic minority in the state.

Figure 2a: Model-generated 2000 district plan for the Greater Chicago Area. The solid black lines indicate the Senate district boundaries, the red lines indicate the House district boundaries.


Our model-generated plan using the 2000 data, on the other hand, also has eight black majority-minority Senate districts, but it has five Hispanic majority Senate districts, one more than the actual plan had. Likewise, as in the actual plan, the model could generate 18 black majority-minority districts, but two more Hispanic majority HRDs, a significant increase from eight in the actual plan. If the number of districts were to reflect the ratio of Hispanic population in the state, the number of Hispanic majority SSDs should have been seven and HRDs should have been 14. If it were posible, the model could configure those 'right' number of districts, but spatial dispersion of the Hispanic population in the state does not allow finding larger numbers than we have found. Despite this, the model plan represents a significant improvement in terms of the proportion of Hispanic majority-minority districts both in the Senate and the House, which was increased from 6.8 percent in the actual plan to 8.5 percent in the model plan.

A concern when using compactness as a criteria for districting is that 'packing' may occur if a minority population is spatially segregated. This will be the case, for instance, when a very large percentage of a district is comprised of one group, which would negatively affect the representation of

Figure 2b: Actual district plan for the Greater Chicago Area

that minority group if voting behavior is racially biased. Having a very large percentage of a minorty group in a district means that the group is assured to elect their candidate of choice in that district, however the votes above the 50 percent needed to win are essentially wasted votes that could have been better used in another district if district boundaries were drawn differently. Our model-generated districting plan shows that consideration of compactness in districting would indeed pack in the case of Illinois. When minority districting criterion was not employed, the model generated four black majority-minority Senate districts and two Hispanic majority-minority House districts.

## Districting with the 2010 Data

Using the 2010 census data, the model-generated the districts maps are shown in Figures 3a, 3b, and 4. The first two show the boundaries of both state Senate and House of Representative districts in the Chicago area while Figure 4 shows the district configurations for the rest of Illinois. Figures 3 a and 3 b also show the majority-minority districts, both African-American and Hispanic, all of which are located in the Chicago area. As mentioned earlier, the state population increased from 12.4 million to approximately 12.8 million. The proportion of the black minority population increased slightly from 14.4 to

Figure 3a: Model-generated 2010 majority-minority Senate districts in the Greater Chicago Area. The solid lines indicate the district boundaries, black-shaded districts are black-majority minority districts, red-shaded districts are Hispanic-majority districts.

14.5 percent, but a significant increase has occurred in the population share of the Hispanic minority, from 12.3 percent in 2000 to 15.8 percent in 2010. Therefore, if the proportion of majority-minority SSDs and HRDs were to reflect the minority population shares in the state, the number of majority-minority Senate districts should be eight and nine, respectively, for the African-American and Hispanic minorities. The corresponding numbers for HRDs should be 17 and 18 , respectively. The largest number of Senate black majority-minority districts the model was able to generate was equal to eight, as desired, but the number of HRDs was 15 , which is somewhat under-representing the black population share. On the other hand, the largest number of Hispanic majority-minority Senate districts was again five and the number of HRDs was 13, which imply considerable underrepresentation of the Hispanic population (particularly the Senate minority districts). Once again, this is a result of the spatial dispersion of Hispanic population in the state which does not allow configuring a few more Hispanic majorityminority districts.

Figure 3b: Model-generated 2010 majority-minority House districts in the Greater Chicago Area. The solid lines indicate the district boundaries, black-shaded districts are black-majority minority districts, red-shaded districts are Hispanic-majority districts.


In most cases, two majority-minority HRDs were found in a majority-minority SSD. In a few cases, only one of the two HRDs in a SSD was a minority district, while in a few other cases a majority-minority HRD could be found in a 'regular' SSD. All of the black and Hispanic majority-minority districts, both for the state Senate and House, were in the Chicago area, only one Hispanic-majority HRD was found in the western suburbs (near Aurora).

Besides configuring majority-minority districts, representation of a minority group may also be affected by the population share of that minority group in a 'regular' district even if it does not exceed the 50 percent cutoff level. This may happen if that minority community can form a coalition with another community in that district with the next largest population share. We did not include an explicit mechanism in the model to take this factor into account. Instead we expost evaluated the population shares of African-American and Hispanic communities in the districts that were not majority-minority. We found that in two model-generated Senate districts the African-American population ratio was

Figure 4: Model-generated 2010 State Senate and House districts for the rest of the State. The black lines indicate the district boundaries, red lines indicate the House district boundaries. The gray lines indicate county lines.

high enough ( 35.6 percent and 40.8 percent) to reach out to the next largest community in those districts to elect either their preferred candidates or strongly influence the election results. We call these 'cross-over districts'. Also, in two other Senate districts there is significant black population (23.0 and 23.1 percent). We call these districts 'influence districts.' Likewise, four of the non-minority Senate districts have a significant Hispanic minority, ranging between 30.1 and 33.0 percent, which would make those communities very influential in the election results. On the House side, there are three cross-over districts for blacks (with 35.6, 40.8 and 45.8 percent population shares) and three for Hispanics (with $37.5,37.8$, and 44.4 percent population shares). Finally, although a somewhat significant Asian population exists in the state to form a House district, no Asian-majority minority HRD could be configured by the model, this is because of the relatively sparse concentration of Asian communities, which are less segregated than the two other minority groups. However, the Asian population shares were significant in two HRDs, 22.1 and 24 percent, which could make those communities politically influential in those districts.

## Concluding remarks

This chapter presents a systematic and automated computer modeling approach to the political redistricting problem considering four basic principles of districting, namely population equality, contiguity, compactness, and community integrity in the form of minority representation. This approach uses only the population data and spatial/demographic characteristics of the data, without consideration of any voting behavior or subjective motives. The entire computational process is programmed considering the criteria specified and once the dataset is input to the computer and the program starts to run, no human interaction (interference) is involved until the computer puts out the solution. Thus, the process is completely free of partisan interests and subjective biases.

The model developed in this study has been applied to two data sets, namely the 2000 and 2010 census data, both for the state Senate and House of Representatives districting in Illinois. The empirical results were highly satisfactory. First, the model-generated districting plans exhibit substantially improved compactness compared to the existing district configurations. Furthermore, in both Senate and House districting, the maps generated by the model notably increased the representation of Hispanic minority in Illinois. Given these, we can safely claim that a systematic districting procedure using the methods presented here could generate superior maps compared to the maps that are drawn by use of other methods, particularly maps created by map-making skills.

The modeling methods used in this study are highly
sophisticated and require a significant level of expertise particularly in mathematical programming modeling, computer programming, and using GIS software. While experts with such skills may not be too many, they are not rare either. Furthermore, the generic nature of the optimization and GIS software employed in this districting study makes it possible to apply these methods to any political redistricting problem in any state, whether it is congressional or legislative districting. Our approaches would be particularly useful when community integrity (minority representation) is of major concern. In that respect, the present study is unique in the political redistricting literature.

A widely observed misconception about mathematical approaches and computer modeling of decision problems like this is the 'fear' of replacement of actual decision makers with a computer program and analysts/modelers. The political districting problem is a political process, thus legislators, governors, and other political entities (e.g. parties) must be directly involved in that process. The purpose of modeling is actually to facilitate this process rather than replace the human decision makers. Political and other real world considerations can be incorporated into the model with involvement of the stakeholders and by incorporating necessary details prevalent in the real world districting. If, for instance, two units must be grouped together in the same district, because of certain cultural or business relationships (such as maintaining township integrity or putting two spatial units in the same district if employees live in one and businesses are located in the other), it is a straight forward matter to include such considerations in the model. Moreover, the solution of a redistricting problem depends on what considerations are to be included, thus it is almost always not unique and equally good/preferred solutions may be possible. One may generate as many district configurations as needed, possibly with the involvement of legislators and stakeholders, and the choice of the final solution can be left to the actual decision makers (i.e. the state legislative body).


# Gerrymandering, Demographic Characteristics, and Voting Preferences 

 Nathan B. Anderson, Robert Kaestner, and Hanqing QiuBroadly defined, gerrymandering is the drawing of electoral districts in non-random, partisan ways so as to advantage the party charged with drawing electoral districts, which in most cases is the majority (incumbent) party. For example, it is often thought that gerrymandering is used to create electoral districts that produce both more and safer seats for the majority party than is justified by voters' preferences. That many electoral seats are safe, in that districts' elections are won by large margins, appears to lend credence to the existence of gerrymandering. Also appearing to lend credence to the existence of gerrymandering is the prevalence of odd -shaped, electoral districts characterized by tentacles that stretch out presumably to include voters sympathetic to a particular candidate or party and to exclude less sympathetic voters. Odd-shaped electoral districts and the many safe electoral seats have led some to advocate for the drawing of electoral districts via a random, nonpartisan process. These advocates argue that a nonpartisan map would produce more competitive elections and election winners that more closely resemble voters' preferences.

However, odd-shaped electoral districts and safe seats do not, in and of themselves, provide evidence of gerrymandering. This is because funny-shaped districts and safe seats may still exist in the absence of gerrymandering. The sorting of people by income, race, and family structure, which all shape voter preferences, into the same neighborhoods, towns and cities implies that voters are not randomly scattered throughout a state. This sorting of voters may make it impossible to draw a nonpartisan (i.e., random) electoral map without safe seats or a large number of seats won by a particular political party. Further, although the ideal is to create compact (e.g., square or rectangular) districts, the inherent difficulty of the redistricting process, which mandates creating a set number of equal-population, contiguous districts that adhere to the

Voting Rights Act of 1965, may make it impossible to create a nonpartisan electoral map without some funny-shaped districts.

In this brief report, we present a method for identifying and measuring the extent of gerrymandering at the congressional district level. Our approach is based on constructing a counterfactual map of congressional districts that is free from partisanship and gerrymandering. As there is no one counterfactual map of congressional districts, we construct a distribution of counterfactual maps of congressional districts, each of which is drawn without reference to political objectives. Specifically, we generate hundreds of congressional district maps for a given state that are constructed using only two, legally required criteria: equal population and contiguity. These provide a sample of congressional district maps from the large number of possible nonpartisan, randomly-drawn electoral maps that could be constructed.

We use two computer algorithms to generate our nonpartisan congressional district maps. Each algorithm separately generates 100 nonpartisan congressional district maps. The only criteria for both methods are equal-population and contiguity. Differences in the algorithms, however, result in one method producing congressional districts that are more compact, as measured by the Polsby-Popper index, than districts produced by the other method.

We use this sample of nonpartisan, randomly-drawn congressional maps to generate an "electoral map" distribution of socioeconomic and demographic characteristics. We then compare the distributions of demographic and socioeconomic characteristics from the sample of nonpartisan, randomly drawn maps to the distribution of demographic and socioeconomic characteristics of an actual congressional
district map. For example, we compare the actual number of congressional districts in a state with greater than 40 percent black population to the (expected) number of congressional districts with greater than 40 percent black population from the sample of nonpartisan, randomly drawn congressional districts of that state. We also use these demographic and socioeconomic characteristics to predict district voting propensities, for example, the proportion of a district expected to vote Democratic, to generate an "electoral map" distribution of voting preferences. Based on these predicted voting preferences, we calculate the distribution of congressional districts that are predicted to be Democratic ( $\geq 50$ percent Democratic vote) and compare this political distribution to the number of Democratic districts in an actual congressional district map.

## An Illinois Example

The 2001 Illinois congressional redistricting process used year 2000 census data to create 19 electoral districts. Based on these same census data, we use each of our two computer algorithms to separately generate 100 nonpartisan Illinois congressional districts maps, each containing 19 equalpopulation and contiguous districts.

Figure 1 displays, alongside the actual congressional district map, one of the 100 Illinois nonpartisan maps generated by what we refer to as Method 1. This method produces districts
that are, on average, more square and rectangular in shape (i.e., more compact) than the actual districts. Similarly, Figure 2 displays one of the 100 Illinois nonpartisan maps generated by what we refer to as Method 2. This method produces districts that are, on average, less compact than the actual districts.

Our objective is to compare the distributions of socioeconomic, demographic and voting propensities of the nonpartisan congressional maps to these same characteristics of the actual congressional map. The first step of this process is to summarize the characteristics of the nonpartisan districts that we generated. For each method, we created 100 congressional district maps with 19 districts each, for a total of 1,900 districts. For each district, we calculated the percentage of the district's population that is black, Hispanic, at least 65 years old, married, does not have a high school education, has at least a bachelor's degree, has annual household income less than $\$ 20,000$, or has annual household income greater than $\$ 75,000$. In addition, we calculate the per-capita household income in each district and use the Polsby-Popper Index (PPI) to calculate the compactness of each district. We selected these nine characteristics because they are associated with individuals' voting behavior and we use the PPI because it is a widely used compactness measurement. These nine characteristics are illustrative examples of characteristics associated with voting behavior and do not represent an exhaustive list of such characteristics.

Figure 1: Illinois' 108th Congressional Districts Map v. An Example of Method 1 Map


Figure 2: Illinois' 108th Congressional Districts Map v. An Example of Method 2 Map


Thus, each of our 100 nonpartisan maps has 19 values, corresponding to the 19 districts of each map, of each of the nine characteristics and the PPI. The second step of the process is to calculate the 25 th, 50 th, and 75 th percentile of the distribution of each characteristic in each of the 100 congressional maps. Then we calculate for each characteristic, separately for each method, the average over the 100 maps of these three percentiles. Because Method 1 and Method 2 produce quantitatively similar results, in the remainder of this report we focus on our Method 2 results.

Table 1 reports these average percentiles for Method 2. The first row reports the results for the percentage of a district's total population that is black. The table shows that, on average, blacks represent less than 5 percent of total district population in 25 percent of a nonpartisan map's districts, less than 8 percent of total district population in 50 percent of a nonpartisan map's districts, and less than 19 percent of total population in 75 percent of a nonpartisan map's districts. Although we do not show it, the 90th percentile is particularly interesting for Illinois because it represents approximately two congressional districts and given that the black population in Illinois was approximately 15.1 percent in 2000, the Voting Rights Act would imply that Illinois have at least two districts with 40 percent or more minority. On average, this is in fact the case in the distribution of nonpartisan maps. Similarly, the sixth row of Table 1 reports the percentage of a district's households that have household

income greater than or equal to $\$ 75,000$. On average, these high-income households represent less than 18 percent of households in 25 percent of a nonpartisan map's districts, less than 24 percent in 50 percent of a nonpartisan map's districts, and less than 36 percent in 75 percent of a nonpartisan map's districts.

In Table 2 panel A , for each characteristic we calculate the share of districts in the actual 108th congressional district map of Illinois that fall within the four quartiles of the distribution of the random, nonpartisan congressional district maps of Illinois. Row One shows that 21 percent of actual districts ( 4 of 19) had a black population of less than 5 percent, 32 percent of districts ( 6 of 19) had a black population of between 5 and 8 percent, 26 percent of districts ( 5 of 19) had black population between 8 and 19 percent, and 21 percent ( 4 of 19) had black population more than 19 percent of their population. In Panel B of Table 2, we calculated the average percentage of districts in the nonpartisan maps that fall within each of the four ranges. These are not all 25 percent because it's not possible to divide 19 districts into four quartiles of an equal number of districts. The standard error is reported below each mean. When comparing the actual map to the nonpartisan maps, we compare the percentage of actual districts within a range to the percentage of nonpartisan districts we expect in that range.

Table 1: Percentile Values of Nonpartisan Maps: Method 2

|  | Average Value at Percentile |  |  |
| :--- | :---: | :---: | :---: |
| Districts' Characteristics | 25th | 50th | 75th |
| \% Black | 0.05 | 0.08 | 0.19 |
| \% No schooling to 12th grade, no diploma | 0.15 | 0.17 | 0.22 |
| \% with Bachelor's degree or more | 0.18 | 0.23 | 0.32 |
| \% Hispanic | 0.04 | 0.10 | 0.17 |
| \% with HH income Less than \$20,000 | 0.12 | 0.20 | 0.24 |
| \% with HH income \$75,000 or more | 0.18 | 0.24 | 0.36 |
| \% Married | 0.41 | 0.46 | 0.48 |
| \% with age 65+ | 0.10 | 0.12 | 0.14 |
| Per-capita income | $\$ 19,017$ | $\$ 21,701$ | $\$ 27,342$ |
| Polsby-Popper Index (compactness) | 0.04 | 0.07 | 0.10 |

Table 2: Distribution of Districts' Characteristics in Actual Map and Nonpartisan Maps [Method 2] (standard errors in parentheses)

| Districts' Characteristics | Panel A: Actual 108th Congressional <br> Districts: \% within each percentile range |  |  |  | Panel B: Average Nonpartisan Map: \% within each range |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | [0,25] | $(25,50]$ | $(50,75]$ | $(75,100]$ | [0,25] | $(25,50]$ | $(50,75]$ | $(75,100]$ |
| \% Black | 0.21 | 0.32 | 0.26 | 0.21 | $\begin{gathered} 0.24 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.27 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.26 \\ (0.01) \end{gathered}$ | $\begin{aligned} & 0.23 \\ & (0.0) \end{aligned}$ |
| \% No schooling to 12th grade, no diploma | 0.32 | 0.37 | 0.16 | 0.16 | $\begin{gathered} 0.23 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.27 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.27 \\ (0.01) \end{gathered}$ | $\begin{aligned} & 0.23 \\ & (0.0) \end{aligned}$ |
| \% with Bachelor's degree or more | 0.26 | 0.32 | 0.11 | 0.32 | $\begin{gathered} 0.23 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.27 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.25 \\ (0.01) \end{gathered}$ | $\begin{aligned} & 0.24 \\ & (0.0) \end{aligned}$ |
| \% Hispanic | 0.26 | 0.37 | 0.11 | 0.26 | $\begin{aligned} & 0.24 \\ & (0.0) \end{aligned}$ | $\begin{gathered} 0.26 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.27 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.23 \\ (0.01) \end{gathered}$ |
| \% with HH income Less than \$20,000 | 0.21 | 0.26 | 0.32 | 0.21 | $\begin{gathered} 0.24 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.26 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.27 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.24 \\ (0.01) \end{gathered}$ |
| \% with HH income \$75,000 or more | 0.21 | 0.32 | 0.26 | 0.21 | $\begin{gathered} 0.24 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.26 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.27 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.24 \\ (0.01) \end{gathered}$ |
| \% Married | 0.21 | 0.26 | 0.26 | 0.26 | $\begin{aligned} & 0.24 \\ & (0.0) \end{aligned}$ | $\begin{gathered} 0.26 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.26 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.24 \\ (0.01) \end{gathered}$ |
| \% with age 65+ | 0.26 | 0.16 | 0.37 | 0.21 | $\begin{aligned} & 0.23 \\ & (0.0) \end{aligned}$ | $\begin{gathered} 0.26 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.28 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.24 \\ (0.01) \end{gathered}$ |
| Per-capita income | 0.26 | 0.21 | 0.37 | 0.16 | $\begin{aligned} & 0.23 \\ & (0.0) \end{aligned}$ | $\begin{gathered} 0.27 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.26 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.24 \\ (0.01) \end{gathered}$ |
| Polsby-Popper Index (compactness) | 0.05 | 0.05 | 0.05 | 0.84 | $\begin{gathered} 0.24 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.26 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.26 \\ (0.01) \end{gathered}$ | $\begin{gathered} 0.24 \\ (0.01) \\ \hline \end{gathered}$ |

Figure 3: Nonpartisan v. Actual: Distribution of Black Population Across Districts


Figure 3 illustrates the comparison between the actual and randomly drawn congressional districts for the percent of population that is black. That the percentage of districts in the highest range is lower for the actual map than the nonpartisan map indicates that the actual map contains fewer districts with blacks representing more than 19 percent of the population than expected in a nonpartisan map. That is, although the actual map contains three districts that are more than 40 percent black, in comparison with the average nonpartisan map, it contains one fewer district that is more than 20 percent black. The small standard error on the nonpartisan percentage implies that the difference between maps is statistically different from zero. Figure 4 compares the actual map and the nonpartisan map in terms of per-capita income. That only 16 percent of districts in the actual map but 24 percent (CI 22 to 26 ) of districts in the nonpartisan map have per-capita income greater than $\$ 27,342$ indicates that the actual map disperses high-income individuals across districts more than the average nonpartisan map.

Finally, we use the socioeconomic and demographic characteristics of each nonpartisan district to predict the propensity to vote Democratic (i.e., share of Democratic voters in each district). To predict the share of Democratic voters we estimate an ordinary least squares regression in which the dependent variable is the share of the district that voted Democrat in the 2002 congressional election, and
the independent variables are the nine socioeconomic and demographic characteristics described earlier. Specifically, we estimate the following:

$$
\begin{aligned}
& \% \text { Dem }_{j}=a+b_{1}\left(\% \text { Black }_{j}\right)+b^{2}\left(\% \text { NoSchool }_{j}\right)+b_{3}\left(\% \text { Bachelor }_{j}\right) \\
& +b_{4}\left(\% \text { Hispanic }_{j}\right)
\end{aligned}
$$

This regression has 19 observations and, in Table 3 (on page 31), we report the estimated coefficients. We use these estimated coefficients and the characteristics of our nonpartisan districts to calculate (predict) the expected Democratic vote share in every nonpartisan district and the expected number of Democratic seats (Democratic share $\geq 50 \%$ ) in every nonpartisan map. Note if we selected a different set of variables then our regression results may change and the distribution of the expected Democratic vote share will change as well. Thus, these regression results are illustrative of our methods, but are not definitive.

Figure 5 (on page 32) presents a histogram of the distribution of expected Democratic seats of the nonpartisan maps. In 38 of 100 nonpartisan plans, we expect eight Democratic seats and in 56 of 100 maps, we expect eight or fewer Democratic seats. That the actual map produced 9 Democratic seats in 2002 implies only a small difference in election outcomes between the actual map ( 9 seats) and what we expect from the nonpartisan map ( 8 seats). In other words, there is a 30

Figure 4: Nonpartisan v. Actual: Distribution of Per-Capita Income Across Districts

percent chance that a nonpartisan map would have contained the same number of Democratic seats as the actual map, which implies only a small effect of gerrymandering on the number of Democratic seats.

It is interesting to note that, although neither computer algorithm incorporated the requirements of the Voting Rights Act, both methods generated nonpartisan congressional maps that, on average, conform to the Voting Rights Act. Specifically, as we show in Table 4 (on page 32), consistent with the Voting Rights Act, on average, a nonpartisan map contains two districts that are more than 40 percent AfricanAmerican. Because Hispanics were 12.3 percent of the Illinois population, the Voting Rights Act implies that there should be at least two districts that are at least 40 percent Hispanic. The actual map contains only one such district and only 41 percent of the nonpartisan maps include one such district, with the remaining 59 percent containing no such districts. The difficulty of drawing maps with more than one Hispanic district reflects the fact that the Hispanic population is less geographically concentrated than the African-American population in Illinois.

## Conclusion

Gerrymandering is widely viewed to exist and evidence to support that belief often consists of illustrations with oddshaped electoral districts and references to the large number of safe seats. However, odd-shaped electoral districts and safe seats may result naturally from the geographic distribution of voters combined with the legal requirements of drawing electoral districts. We have presented here a method for differentiating between gerrymandering and naturally occurring consequences of redistricting that may appear to be gerrymandering. We presented an example of our approach using Illinois congressional districts.

Our approach is relatively straightforward. Using two different computer algorithms, and only two legally required criteria, equal population and contiguity, we constructed hundreds of congressional district maps for Illinois (i.e., 19 district plans). These maps were randomly constructed and were created without regard to partisan objectives, or any objectives other than equal population and contiguity. Moreover, we used algorithms that differed significantly in the compactness of the constructed congressional districts. These random congressional district maps can be viewed as a subset (non-random) of all possible nonpartisan, randomly drawn congressional district maps. As such, they are a useful,

Table 3: Districts' Characteristics Used to Predict Districts' Share Democratic Votes

| $Y=$ Share Democrat [0, 1] <br> Independent Variables | Coefficient <br> (std error) |
| :--- | :---: |
| Black | -0.347 |
|  | $(0.592)$ |
| Low Income | 3.468 |
|  | $(4.594)$ |
| High Income | 5.785 |
|  | $(3.527)$ |
| Hispanic | 0.506 |
|  | $(2.002)$ |
| Age 65+ | 7.900 |
|  | $(3.208)$ |
| No School | -0.685 |
|  | $(5.132)$ |
| Bachelors | -2.412 |
|  | $(2.488)$ |
| Married | -5.954 |
|  | $(3.100)$ |
| Per-Capita Income | -0.024 |
|  | $(0.043)$ |
| Constant | 1.306 |
| Observations | $(1.792)$ |

if not perfect, benchmark that can be used to assess deviations of the actual congressional districts (108th Congress) from a random set of congressional districts.

Using these nonpartisan random maps, we calculated the distribution of selected socioeconomic and demographic characteristics. We then compared the distribution of these socioeconomic and demographic characteristics to the distribution of these characteristics in the actual congressional district map of Illinois. For example, we showed that the actual Illinois congressional district map had fewer districts with a relatively large proportion of high-income ( $\$ 75,000$ or more) households and low-educated (<High School Degree) persons than did the randomly drawn congressional district maps. There were also statistically different distributions of race and ethnicity between the actual Illinois map and the randomly drawn congressional district maps.

We also presented an assessment of potential voting differences between the actual Illinois map and the randomly drawn maps. While this analysis is dependent on the socioeconomic and demographic variables we selected to examine, and the previous election we used to develop weights to predict voting propensities for given
socioeconomic and demographic characteristics, it is illustrative of the extent to which gerrymandering can influence elections. In our example, there was only moderate evidence that gerrymandering affected voting in 2002; the nine Democratic districts in 2002 election occurred in 30 percent of the randomly drawn districts.

Overall, we have presented a novel way to identify the existence and extent of gerrymandering. Our approach can be easily applied to every state and every year because of the efficiency of the computer algorithms we developed. Our approach can also be used at any electoral level, although at more disaggregated levels (e.g., state representatives), the computing time will increase. An important contribution of our approach is its ability to be applied to different time periods and all states. Thus, we can track how the extent of gerrymandering evolves over time and across states and link these changes to the political party that drew the electoral map.

Figure 5: Nonpartisan v. Actual: Distribution of Expected Number of Democratic Districts


- Nonpartisan Maps

Table 4: Nonpartisan Maps, Actual Map, and the Voting Rights Act

|  | Nonpartisan Plans |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Districts with at least $\mathbf{4 0 \%}$ of population that is: | $\boldsymbol{\operatorname { m i n }}$ | $\max$ | avg |  |
| Actual Map |  |  |  |  |
| Hispanic | 0 | 1 | 0.41 | 1 |
| Black | 1 | 3 | 1.83 | 3 |

## 10

Legislative Redistricting in the 50 States
in 2010: Best Practices Christopher Z. Mooney

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As sure as summer follows spring, the national census at the beginning of each decade leads to that most obscure, most political, but most important political event: the redrawing of legislative district boundaries, or redistricting. The United States Constitution gives each state the responsibility to draw its state legislative and congressional districts' boundaries, and the U.S. Supreme Court requires that they do so again each decade so that the districts represent the population shifts of the previous 10 years. ${ }^{1}$ Where these boundaries are drawn can have a great effect upon the make-up of these legislative bodies and, thereby, the public policies that they adopt.

Of course, the impact of redistricting is marginal - no amount of map manipulation is going to result in a Democratic majority in the Idaho Senate, for example. But because politics is a game of margins, changing the partisan outcomes of a few districts can sometimes have significant effects, at both the national and state levels. In addition, the placement of every legislative district boundary can have life-anddeath impact on the political careers of many politicians and political hopefuls. Not only do the boundaries determine the voters that someone running for the legislature or Congress must face, they also influence which specific opponents he/ she will face. Thus, at both the macro and micro levels, it is difficult to overstate the political and policy significance of the decennial legislative redistricting on which the states will embark in 2011.

[^7]The procedures that a state uses to determine the placement of its legislative district boundaries may affect where those lines are drawn and, thus, can have important political and policy consequences. This chapter describes and evaluates the redistricting criteria and mechanisms that the 50 states use in redistricting. This assessment may assist policymakers and citizens in Illinois in their efforts to reform the Prairie State's redistricting process.

## Why Re-District?

In each state, every member of the legislature and of the state's delegation to the U.S. House of Representatives is elected from a specific, legally defined, geographical subsection of the state - his or her legislative district. The boundaries of these districts are described precisely in state statutes or regulations. ${ }^{2}$ They run down specific streets and across specific fields, golf courses, airports and so forth (rather than through houses and apartment buildings), so that each square foot of the state is placed into one and only one state senate, one state house, and one U.S. House district. ${ }^{3}$

Legislators typically invest considerable time and effort into

[^8]building goodwill and favorable name recognition in their districts through campaigns, newsletters, public appearances, personal favors, pork-barrel projects, and professional service. So like with many other things about elections, legislators generally do not like their district boundaries to change. And because of this, before the 1960s, legislative districts often went unchanged for many decades.

The problem with failing to change legislative district boundaries is that the states' populations are constantly shifting. In particular, the early- to mid-20th century was a time of great population upheaval and change, with people being driven from the countryside to the cities by the Great Depression, two World Wars, and the mechanization of agriculture. Because of this migration, districts that may have been equal in population in 1900 were very unequal by 1960. Some states suffered from extreme legislative malapportionment, that is, unequal numbers of people living in different districts. For example, Connecticut's districts were so malapportioned in 1960 that a party controlling districts containing as little as 12 percent of the state's population could have had a majority of seats in its state House. ${ }^{4}$ Ironically, the worse malapportionment got, the more politically difficult it was for state policymakers to do anything about it because those lawmakers and voters who benefited from it had a strong incentive not to change it. Lawmakers sometimes even ignored their own state laws requiring redistricting because the political pain was just too great. ${ }^{5}$

In a series of landmark decisions beginning in 1962, the U. S. Supreme Court changed all this by ruling that the Equal Protection Clause of the 14th Amendment to the Constitution required that districts in the same legislative chamber had to be "substantially equal" in population. ${ }^{6}$ In these cases, the Court established the principle of "one person, one vote," holding that all votes for seats in a chamber must be of equal value. With malapportioned districts, a person's vote in a smaller district is worth more than a person's vote in a larger district. For instance, a person in a district of 5,000 people would have three times the influence in an election of a person in a district of 15,000 people for the same chamber $-1 / 5,000$ versus $1 / 15,000$. And influence in an election is influence in the legislature.

[^9]As a result of these Supreme Court decisions, the states spent the rest of the 1960s undertaking the gut-wrenching task of redrawing their often heavily imbalanced state legislative and congressional districts so that they would be equal in population (based on the 1960 U.S. Census). In most states, the legislature was required to draw its own districts, so the process was particularly painful because it was clear that these changes would cause many legislators to lose their next re-election efforts. Where legislatures were not able to get the job done in a timely manner, the courts stepped in. For example, when Illinois' legislature and governor failed to agree on a set of districts for its lower chamber in 1964, a federal judge forced the issue in one of the state's most infamous political episodes - the "Bed-Sheet Ballot" election, with all candidates running at-large. ${ }^{8}$ All state voters were faced with a 33 -inch paper ballot on which each of them had to choose from among 236 candidates for 177 state house seats. This political embarrassment forced lawmakers to draw legally acceptable districts by 1966. One way or another, by 1968, every state had redrawn its legislative and congressional districts to comply with the Supreme Court mandates.

But because Americans are always on the move, equalpopulation districts do not stay equal for long. The Supreme Court requires that states go through the redistricting process after each national census to adjust for population shifts that occurred over the previous decade. But if redistricting in the 1960s was like a political hurricane, the regular decennial redistricting is generally only like a really bad thunderstorm. It still causes a major political battle in every state every decade, but policymakers have developed the processes and skills required to fight those battles in a relatively orderly way. The result is that the changes made each decade are not nearly as large as those that were required in the 1960s, when policymakers had to make up for generations of neglect.

## Drawing Legislative Districts

Redistricting in the United States today hinges on three dimensions: its politics, the criteria policymakers use in drawing legislative maps, and the mechanisms states use to adopt their final maps.

## The Politics of Legislative Redistricting

Three general forces shape the politics of legislative redistricting:

- Countervailing interests among those involved in drawing the maps (especially rank-and-file state legislators, legislative leaders, and political parties);

[^10]- The general public's lack of concern with, and lack of knowledge of, the redistricting process; and
- No consensus on the criteria for assessing legislative maps.

The politics that follow from these three forces yield districts that have two general characteristics.

First, most new legislative districts tend to be electorally safe for legislators serving when they are drawn. Most state legislatures have a significant role in determining these districts (see below), and each lawmaker has a strong and direct interest in precisely how his/her district is drawn, probably more so than any other single piece of legislation in the entire decade. Because they determine which voters and even which opponents a lawmaker will face for reelection, these boundaries can make or break political careers. So the price of a legislator's vote for an overall districting plan may be a favorable district for him/herself. As a result, most redistricting plans contain many districts with lopsided partisan balances incorporating as much of an incumbent's previous district as possible. While this effect is strongest when both parties have a voice in the process due to divided government or the use of a bipartisan redistricting commission, it happens in many districts even when a single party controls the process (see below). ${ }^{9}$ This incumbentprotection dynamic mutes the potential for redistricting to shake up a state's political system every 10 years.

Second, although less extensive than incumbent protection, when one party controls the process, whether by having unified state government or by controlling a redistricting commission, political gerrymandering can occur. That is, the party in control may try to draw districts so as to improve its chances of winning more seats. A majority party can do this both by dispersing minority party voters so that they are less than a majority in many districts - so-called "cracking." A variant of cracking is "spoking," where long districts are drawn from a solid core of majority-party voters to include some minority-party voters, thereby diffusing their potential to elect a legislator. This technique can be seen in several Illinois state legislative districts in the Chicagoland area (e.g., House Districts 36 and 28 and Senate District 8). The Democrats who controlled the state's redistricting commission in 2001 sent out district tendrils from predominantly Democratic Chicago and its close-in suburbs into the predominantly Republican areas farther out.

Another approach to gerrymandering is "packing," placing large numbers of minority-party voters into a few seats.
${ }^{9}$ John N. Friedman and Richard T. Holden, "The Rising Incumbent Reelection Rate: What's Gerrymandering Got to Do with It?" Journal of Politics 71(2009):593-611.

Because every vote over 50 percent is essentially "wasted" because it is not needed to win the seat, constructing districts of 70 percent or more of minority-party voters means that there are fewer of them to make more competitive districts elsewhere.

While this may make the minority-party legislator in such a district very happy since he/she will be electorally safe, it limits the party's overall percentage of seats in the chamber. The 2001 Illinois legislative map demonstrates this technique in downstate cities, where Democrats packed Republicans into House and Senate seats surrounding the cities' cores to maximize their own relatively small numbers there. Sometimes this strategy was successful (as in Senate Districts 52 and 46), while elsewhere it was not (as in House District 99).

## Redistricting Criteria

Aside from political considerations, what criteria do policymakers use in deciding exactly where to place the boundaries for each legislative district? Some redistricting criteria are codified or based on legal interpretation, while some are merely custom and preference; some criteria apply equally to all states, while some apply to some states but not others. ${ }^{10}$

Three criteria established by the Supreme Court apply in all states.

First, as noted, the Court requires that all districts for a given chamber be substantially equal in population, but this is a criterion that is much stricter for congressional districts than for state legislative districts. The Court has held that the largest and smallest U.S. House districts can be no more than 1 percent different in population, while it allows the maximum deviation for state legislative districts to be as much as 10 percent. ${ }^{11}$ However, most states, including Illinois, make their legislative districts much closer than this.

Second, redistricters in every state must also abide by evolving Supreme Court decisions regarding race. Before the 1980s, the Court overturned efforts to pack and crack racial minorities to dilute their political power, but since the 1990s, it has held that race could not be an overriding redistricting criterion to increase minority representation. In general

[^11]today, race cannot be an "overriding, predominant force" in the redistricting process. ${ }^{12}$

Third, each legislative district in every state must be contiguous, that is, all of its landmass must be touching (except for islands). But perhaps surprisingly, the Supreme Court has held that it is constitutional for those drawing legislative districts to pursue partisan advantage, so long as they break no other state or federal laws in the process. ${ }^{13}$

In addition to these three universal requirements, there are other potential redistricting criteria that a state might use in drawing its maps. States differ in their use of these other criteria, and they may be mandated or merely desired. Because different criteria may conflict in developing maps, formal or informal priorities for those criteria develop in practice.

Some states even establish legal hierarchies for these criteria, requiring that certain goals be met before others

[^12]are considered. For instance, Arizona requires that, after the three universal criteria are met, districts must first respect communities of interest, then use visible geographic features and existing political boundaries, and finally promote political competition. Such trade-offs are often easier codified than made in practice. The balancing of redistricting criteria usually becomes, like politics itself, the art of the possible.

Table 1 shows where six "traditional districting principles" are codified by state constitutions, statutes, rules, and/ or court cases, for congressional and/or state legislative redistricting. ${ }^{14}$ States define their criteria formally in an effort to enhance those values that have been agreed upon before the political heat of the redistricting season and to reduce partisan gerrymandering. This strategy can be successful, at least sometimes. ${ }^{15}$ Codifying its criteria also helps a state if

[^13]Table 1: Redistricting Criteria in the States

| State | Political Subdivision Preservation | Compactness | Voting Rights Act of 1965 Considerations | Communities of Interest | District Core Preservation | Incumbent <br> Protection Prohibition |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AL | B | B | B | B | B |  |
| AK | L | L |  | L |  |  |
| AR | B |  | B |  | B |  |
| AZ | B | B | B | B |  | B |
| CA | L | L | L | L |  | L |
| CO | L | L | L | L |  |  |
| CT | L |  |  |  |  |  |
| DE |  |  |  |  |  | L |
| FL |  |  |  |  |  |  |
| GA | B |  | B |  | B |  |
| HI | L | L |  | L |  | L |
| ID | B | B | B | B |  | B |
| IL |  | L |  |  |  |  |
| IN |  |  |  |  |  |  |
| IA | B | B | B |  |  | B |
| KS | B | B | L | B | L | L |
| KY | C |  | C | C | C |  |
| LA | L |  |  |  | L |  |
| ME | L | L |  |  |  |  |

Table 1 (con't): Redistricting Criteria in the States

| State | Political Subdivision Preservation | Compactness | Voting Rights Act of 1965 Considerations | Communities of Interest | District Core Preservation | Incumbent <br> Protection <br> Prohibition |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MD | L | L | B |  | $\mathrm{B}^{2}$ |  |
| MA | L |  |  |  |  |  |
| MI | B | B | C |  |  |  |
| MN | B | B | B | B |  |  |
| MS | L | L | B |  |  |  |
| MO | L | B | L | L | L |  |
| MT | L | L | L |  |  | L |
| NE | B | B | B |  |  | B |
| NV | B | B | B | L |  |  |
| NH | L |  |  |  |  |  |
| NJ | L | L | C |  | C |  |
| NM | B | B | B | B | $B^{2}$ |  |
| NY | L | L |  |  |  |  |
| NC | B |  | B |  | C |  |
| ND | L | L |  |  |  |  |
| OH | L | L |  |  |  |  |
| OK | L | L |  | L |  |  |
| OR | B |  | B | B |  | B |
| PA | L | L |  |  |  |  |
| RI |  | L |  |  |  |  |
| SC | B | B | B | B | B |  |
| SD | L | L | L | L |  |  |
| TN | L |  | L |  |  |  |
| TX | L |  |  |  |  |  |
| UT |  | B |  |  |  |  |
| VT | L | L |  | L |  |  |
| VA | $\mathrm{B}^{2}$ | B | B | $\mathrm{B}^{2}$ | $\mathrm{B}^{2}$ |  |
| WA | B | B |  | B |  | L |
| WV | B | B |  |  |  |  |
| WI | L | L |  |  |  |  |
| WY | B | B | L | L |  | L |
| TOTAL | 44 | 36 | 27 | 21 | 13 | 12 |
|  |  |  |  |  |  |  |

${ }^{1}$ Allowed to be used as a criterion, but not required
$\mathrm{C}=$ criteria applied to US House of Representatives districts; $\mathrm{L}=$ criteria applied to state legislative districts; $\mathrm{B}=$ criteria applied to both types of districts; blank = no such criteria for either type of district.

Sources: Criteria found in state constitutions, statutes, and redistricting commission guidelines, as reported in: Jason Barabas and Jennifer Jerit, "Redistricting Principles and Racial Representation," State Politics and Policy Quarterly 4(2004):415-35; Jennifer Jerit, Paul Gottmoller, and Jason Barabas, "U.S. Redistricting Standards: A Protocol for Data Collection," Southern Illinois University-Carbondale (unpublished manuscript, 2003); National Conference of State Legislatures, Redistricting Law 2010 (Denver, CO: National Conference of State Legislatures, 2009).
its districting plan is challenged in court; these criteria can be used to justify the lines it has drawn. ${ }^{16}$ These criteria are:

- County and/or municipal boundaries must be followed to the extent possible ( 44 states). ${ }^{17}$ Legislative boundaries that match other political boundaries with which voters are already familiar enhance accountability and voter awareness and understanding of legislative elections.
- Compactness (36 states). ${ }^{18}$ Establishment of this criterion speaks directly to the complaint that gerrymandered legislative districts can be spindly, oddly shaped pieces of land. Having compact districts reduces cracking (although it may increase packing) and increases the chances that lawmakers live close to their constituents.
- Section 2 of the Voting Rights Act of 1965 (VRA) (27 states). The VRA is a federal law that prohibits racial discrimination in voting procedures, including redistricting. While several states and localities with a history of racial discrimination are targeted specifically in the law, many other states also require their congressional and/or legislative districts to meet the VRA's standards. ${ }^{19}$
- Preservation of "communities of interest" (21 states). ${ }^{20}$ A relatively new legal concept and criterion in legislative redistricting, one definition of communities of interest is "when residents share substantial cultural, economic, political, and social ties." ${ }^{21}$ Preserving a community of interest in a legislative district can enhance representation by making the political interests of a legislator's constituents more homogeneous.
- Preservation of previous district's core (13 states). ${ }^{22}$ When new districts are as similar as possible to old districts, both incumbent lawmakers and voters know each other better, allowing for better informed citizens, better representation, and more accountability.
- Incumbent protection prohibition (12 states). ${ }^{23}$ Policy

[^14]makers establish this criterion in recognition of the conflict of interest for legislators when they have a hand in drawing their own districts. Obviously, this criterion can conflict directly with that of preserving a district's core, although it may mitigate lawmakers' temptation to pack their own partisans into their districts.

These criteria are not the only ones used in redistricting. For example, redistricters in a few states are also required to enhance political competition in legislative and congressional races; this is an informal criterion often desired by redistricting reformers, too. Clearly, most of the traditional criteria are also motivated by a wish to enhance political competition, by reducing techniques of partisan and incumbent-protection gerrymandering.
Redistricting is made more difficult by conflicts between actors in the process who value different criteria and by conflicts between these criteria themselves. Furthermore, the definitions of these criteria are often open to interpretation and their achievement in any given map is usually difficult. In the end, these inconsistencies, conflicts, and difficulties combine to make legislative redistricting one of the most challenging and politically charged policymaking activities undertaken by a state in a decade.

## Redistricting Mechanisms

The details of the mechanisms that state policymakers use to draft and adopt their legislative maps - being guided by the above criteria - vary considerably among the states, but they break down into two major categories and a total of six subcategories, as shown in Table 2 for state legislative redistricting. ${ }^{24}$ This analysis, like that of redistricting criteria, breaks states down by their formal, legal mechanisms. Informal mechanisms, procedures, and norms also vary significantly from state to state.

Thirty-seven states use their state legislative process in some way to develop and adopt their state legislative maps. Of these states, 20 use the full, regular process to do so, including giving the governor veto power over the final product; in Michigan and North Carolina, the governor is left out of the process. Eleven states, including Illinois, give the legislature and governor first crack at redistricting. But if these policy makers miss a given deadline, the states then

[^15]Table 2: Mechanisms for State Legislative Redistricting ${ }^{\text {a }}$

| State | Legislature w/ gubernatorial approval | Legislature as sole authority | Legislative process w/ backup | Legislative process w/ advisory commission | Commission/ <br> Panel- <br> Partisan or bipartisan | Commission/ PanelIndependent |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AK |  |  |  |  | X |  |
| AR |  |  |  |  | X |  |
| AZ |  |  |  |  |  | X |
| CA |  |  |  |  |  | X |
| CO |  |  |  |  | X |  |
| CT |  |  | X |  |  |  |
| DE | X |  |  |  |  |  |
| FL |  |  | X |  |  |  |
| GA | X |  |  |  |  |  |
| HI |  |  |  |  | X |  |
| ID |  |  |  |  | X |  |
| IL |  |  | X |  |  |  |
| IN |  |  | X |  |  |  |
| IA |  |  |  | X |  |  |
| KS | X |  |  |  |  |  |
| KY | X |  |  |  |  |  |
| LA |  |  | X |  |  |  |
| ME |  |  |  | X |  |  |
| MD |  |  |  |  | X |  |
| MA | X |  |  |  |  |  |
| MI |  | X |  |  |  |  |
| MN | X |  |  |  |  |  |
| MS |  |  | X |  |  |  |
| MO |  |  |  |  | X |  |
| MT |  |  |  |  | X |  |
| NE | X |  |  |  |  |  |
| NV | X |  |  |  |  |  |
| NH | X |  |  |  |  |  |
| NJ |  |  |  |  | X |  |
| NM | X |  |  |  |  |  |
| NY |  |  |  | X |  |  |
| NC |  | X |  |  |  |  |
| ND | X |  |  |  |  |  |
| OH |  |  |  |  | X |  |
| OK |  |  | X |  |  |  |
| OR |  |  | X |  |  |  |
| PA |  |  |  | X |  |  |
| RI | X |  |  |  |  |  |

Table 2 (con't): Mechanisms for State Legislative Redistricting ${ }^{\text {a }}$

| State | Legislature w/ gubernatorial approval | Legislature as sole authority | Legislative <br> process <br> w/ backup | Legislative process w/ advisory commission | Commission/ PanelPartisan or bipartisan | Commission/ <br> Panel- <br> Independent |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SC | X |  |  |  |  |  |
| SD |  |  | X |  |  |  |
| TN | X |  |  |  |  |  |
| TX |  |  | X |  |  |  |
| UT | X |  |  |  |  |  |
| VT |  |  | X |  |  |  |
| VA | X |  |  |  |  |  |
| WA |  |  |  |  | X |  |
| WV | X |  |  |  |  |  |
| WI | X |  |  |  |  |  |
| WY | X |  |  |  |  |  |
| TOTAL | 20 | 2 | 11 | 4 | 11 | 2 |

${ }^{\text {a }}$ See Footnote 24 for a discussion of the mechanisms used for redistricting the U.S. House.
Sources: Douglas Johnson, Ian Johnson, and David Meyer, "Redistricting in America: A State-By-State Analysis." Report of the Rose Institute of State and Local Government, Claremont, CA (2010); Michael P. McDonald, "A Comparative Analysis of Redistricting Institutions in the United States, 2001-02," State Politics and Policy Quarterly 4(2004):371-95; National Conference of State Legislatures, Redistricting Law 2010 (Denver, CO: National Conference of State Legislatures, 2009); updated by the author.
move to a formally defined backup plan. The deadline is typically three to six months after the release of the census data (which is typically in February following the census), because the maps need to be in place well before the next primary election. These redistricting backup plans consist of panels of officeholders (or those appointed by officeholders) and/or judges. Regardless of these details, maps adopted by the legislative process under unified government tend to yield partisan gerrymanders, while those adopted under divided government tend to yield incumbent-protection gerrymanders. ${ }^{25}$

The other four states that use the legislative process to adopt their maps begin the process with a formal advisory commission that drafts an initial plan. The extent to which these legislatures follow their commissions' advice varies considerably. For example, on one hand, Iowa's advisory group is the Legislative Services Agency (LSA), the legislature's nonpartisan staff agency, and its report is treated as nearly sacrosanct. While Iowa lawmakers may send the draft plan back to the LSA for revisions, public and media pressure considerably limit the extent that this is done. On

[^16]the other hand, the report by New York's Legislative Task Force on Demographic Research and Reapportionment is typically regarded as just the starting point for extensive discussion and revision of the plan in the legislature itself.

The other major type of redistricting mechanism is a panel or commission that can adopt a set of maps without any direct input or approval from the legislature itself. Redistricting commissions were established in recognition of the conflict of interest that state legislatures have in drawing our own districts. In addition, giving the responsibility for this very complex and technical job to a relatively small commission made more sense to reformers than hashing it out in the legislative chamber with dozens or even hundreds of members. Eleven states' redistricting commissions are either bipartisan or unabashedly partisan, with members being either elected officials or their appointees. Some states require at least one nonpartisan member. But while redistricting commissions originated as good government reforms, the combination of public indifference and lawmakers' intense interest in the process has generally rendered them toothless in terms of limiting gerrymandering and increasing competitiveness. The maps that these commissions produce tend not to be systematically different from those drawn through the legislative process: bipartisan commissions tend to yield incumbent-protection gerrymanders, while commissions
with a partisan slant tend to yield partisan gerrymanders. But while most redistricting mechanisms are unsurprisingly permeated with politics, two states - Arizona and California - have each recently undertaken reforms to develop a redistricting process that is more independent of politics, or at least more independent of the influence of elected public officials. ${ }^{26}$ By initiative in 2000, Arizona voters created the Independent Redistricting Commission (IRC), made up of five private citizens - two Democrats, two Republicans, and one nonpartisan person - selected through a two-step process designed to limit the influence of the legislature. The initiative gave the IRC a clear mandate about redistricting process and criteria, and it gave the IRC the authority to draw and adopt a set of maps without legislative approval. The IRC was thoroughly challenged in the courts during the 2000s, but it stood that test and will be used again in 2011. California had tried many times to reform its redistricting process, a system criticized for producing too much incumbent-protection gerrymandering. But in 2008, voters narrowly approved Proposition 11, establishing the Citizens Redistricting Commission (CRC) charged with drawing and adopting state legislative and Board of Equalization districts. To help encourage political independence, CRC members are selected through a seven-step, Rube-Goldberg-like process that includes private citizens applying for these positions (over 16,000 of them applied in 2010) and a random drawing, among other things. Whether this process works in practice and how its maps will hold up to court challenges remains to be seen. ${ }^{27}$

## Legislative Redistricting Reform in Illinois

Legislative redistricting has been controversial and the target of reformers in Illinois since the 1970 State Constitution was adopted. But taking a comparative-state view, we see that redistricting here has been no more controversial than in other states, especially large and complex states. There are so many cross-cutting interests at stake in deciding the parameters of a decade's worth of legislative and congressional elections, so much uncertainty, and so many potential winners and losers, that intensive disagreements are probably inevitable. Can Illinois' redistricting process be improved? Certainly, but it is important to understand both the lessons of other states' experiences and what is feasible in Illinois for any reform to be successfully implemented. What follows is a modest proposal related to both redistricting criteria and mechanism.

[^17]First, Table 1 shows that Illinois codifies fewer redistricting criteria than most other states. State legislative districts are required only to be compact, and there are no formal criteria at all for congressional districts. While codifying redistricting criteria is certainly no guarantee of achieving them, without such criteria laid out in law or rule, that debate over redistricting is often circular, with opponents talking past one another and pure political muscle, rather than logic or evidence, winning the day. Thus, the first thing that redistricting reform in Illinois needs is not a new mechanism, but rather a clear consensus on the characteristics that Illinoisans think would make for a good legislative map. To date, redistricting reform debate in the state has centered on mechanism, not criteria. It may be easier to achieve political consensus on the latter than the former.

Second, if Illinois policymakers can agree upon criteria to judge a set of legislative maps, they can take up the question of redistricting mechanism. Few are pleased with the state's current system of using the legislative process, backstopped by a bipartisan committee with the random tiebreaker. The system has yielded incumbent-protection gerrymanders when a bipartisan consensus could be achieved (typically, only for congressional redistricting) and partisan gerrymanders when one side was dominant. What mechanism could achieve a better result? This question, again, assumes a criterion that has not been formally agreed-upon - that partisan and incumbent-protection gerrymanders are not desirable. Perhaps the legislative process would yield less problematic maps if clear criteria were codified. Alternatively, the state could move toward a redistricting commission as the primary, rather than the backup, mechanism. But studies have shown that even institutionalizing a typical redistricting commission does not end gerrymandering.

Could Illinois adopt a very independent commission, like those in Arizona and California? Remember that Arizona and California did so through citizen initiative, a process that is rarely used and potentially unconstitutional for this purpose in Illinois. Even if feasible, such a commission would likely yield desirable legislative maps only if it is given a clear mandate of redistricting criteria.


## APPENDIX 1

## An Integer Programming Model for Political Districting

First, we give a brief definition of the concepts and terminology used in the model development. A district is a collection of relatively small indivisible geographical areas within a state, each of which is called a 'base unit' or simply 'unit.' Unit boundaries are specified in advance and are not allowed to change during the districting process. Counties, census tracts, or census blocks are often considered as base units in redistricting studies. A contiguous district allows traveling from any unit to any other unit in that district without crossing another district's boundaries. Population equality requires districts to have 'approximately equal' populations, thus allowing slight differences from the ideal (average) district population. The tolerable inequality limit is specified in state constitutions. Typically, the maximum deviation limit is 5 percent or less for legislative districts, much smaller ( $<1$ percent) for congressional districts. Compactness of a district is measured here by the sum of population weighted distances between all units in that district to a central unit in that district (straight-line traveled distances), similar to the concept introduced by Weaver and Hess (1963). The smaller the total traveled distance, the higher the compactness. Thus, maximizing compactness of a district is equivalent to minimizing the total traveled distance. This approach promotes, but does not necessarily guarantee, spatial contiguity when base units are grouped to form districts, since minimization of the total travel distance would favor grouping units closest to a center. These units would be adjacent to each other and form a contiguous area (for a graphical illustration see Figure 1). Several previous studies used this approach and verified empirically that maximizing compactness generally results in contiguity as well. In this study we introduce an explicit mechanism to guarantee contiguity as will be discussed below. The overall compactness of a districting plan is the sum of compactness measures of the individual districts in that plan (as in Harris, 1964).

The districting problem is stated in mathematical terms as follows. Given a set of base units $i \in I$ and a sufficiently large set of candidate central units, $C \subset I$, select $k$ central units from $C$, where $k$ is the number of districts to be configured, and assign base units $i \in I$ into the selected central units in such a way that: 1) each unit is assigned to one and only one central unit; 2) the total population of units assigned to each central unit lies within the specified upper and lower bounds for individual districts; 3) the sum of population-weighted distances from units to their assigned central units is minimized; 4) the total number of majority-minority districts is minimized. For the latter, we require that at least 50 percent of the population in that district is
comprised by the designated minority population. This problem can be formulated as a linear integer programming model as an instance of the p-median problem extending the model introduced by Hess et al. (1965).

The following notation is used in the algebraic model: $I$ and $C$ are the sets of base units and candidate central units, respectively; $p_{i}$ denotes the population of unit $i$ and $\bar{p}$ is the average district population given by $\bar{p}=\sum_{i} p_{i} / k ; \mathscr{d}_{i c}$ odenotes the population-weighted distance from unit $i \in I$ to central unit $c \in C$, i.e. $d_{i c}^{0}=p_{i} d_{i c}$ where $d_{i c}$ is the distance ${ }^{1}$ from unit $i$ to unit $c$; $X_{i c}$ a binary variable where $X_{i c}=1$ when unit $i$ is assigned to the district centered at unit $c$, and $X_{i c}=0$ otherwise. The binary variables $X_{c c}$ are of special importance since they indicate whether each candidate central unit $c$ is selected as a district center, in which case $X_{c c}=1$, or not. The specification of $I$ is straight forward and depends on the desired spatial resolution of the base units to be considered. Specification of $C$ is a critical issue which will be discussed later.

Suppose $k$ districts are to be formed satisfying the characteristics listed above. The following integer programming model, which will be referred to as Model-A hereafter, determines which of the $k$ central units should be selected and which units should be assigned to each of the selected centers:

$$
\begin{equation*}
\text { Minimize } \quad \sum_{i, c} \mathscr{d}_{i c}^{i c} X_{i c} \tag{1}
\end{equation*}
$$

Subject to:

$$
\begin{align*}
& \sum_{c} X_{c c}=k  \tag{2}\\
& \sum_{c} X_{i c}=1 \text { for all } i  \tag{3}\\
& \sum_{i} X_{i c} \leq m X_{c c} \text { for all } c \tag{4}
\end{align*}
$$

[^18]\[

$$
\begin{array}{ll}
\sum_{i} p_{i} X_{i c} \leq(1+l) \bar{p} X_{c c} & \text { for all } c \\
\sum_{i} p_{i} X_{i c} \geq(1-l) \bar{p} X_{c c} & \text { for all } c \tag{6}
\end{array}
$$
\]

$$
\begin{equation*}
X_{i c}=0,1 \quad \text { for all } i, c \tag{7}
\end{equation*}
$$

The objective function [1] represents the overall compactness of the district plan.
Equation [2] implies that exactly $k$ centers must be chosen from the candidate district centers list so that a district can be formed around each of them. Equation [3] states that every unit must belong to exactly one district (assigned to one of the selected centers), while constraint [4] states that a unit can be assigned to a candidate district center only if the latter is chosen as one of the $k$ district centers (thus a district is formed around it). To see why this is so, suppose $X_{c c}=1$, i.e. candidate unit $c$ is selected as a district center. Then, [4] allows assigning up to $m$ units to the district centered at $c$, where $m$ is a user-specified arbitrarily large number (an overestimate of the number of units that can be included in any district). If $c$ is not selected as a district center, i.e. $X_{c c}=0$, then [4] implies that $X_{i c}=0$ for all $i$, that is no unit can be assigned to $c$, as it should be. Constraints [5] and [6] imply that the total population of all units assigned to any given district has to be within the allowable deviation limits from the ideal district population.

In problems with a small number of base units, each unit may be considered as a potential district center, i.e., $C \equiv I$. This would allow maximum flexibility in terms of location and shape of the districts, which would be the ideal situation. However, even for moderately large number of base units, assuming $C \equiv I$ would lead to a very large number of binary $X_{i c}$ variables (specifically $|I|^{2}$ binary variables, where $|I|$ denote the cardinality of $I$ which may make the model computationally intractable. Therefore, restricting $C$ to a reasonably small proper subset of $I$ is inevitable in most practical applications. Allowing an adequate level of flexibility while maintaining computational tractability is an important matter that affects the objectivity of a district plan generated by the model. We address this issue by using a systematic procedure that involves a second linear integer programming model described below.

For each unit $i \in I$, we generate a minimal compact (circular) district centered at $i$ by solving the following problem:

$$
\begin{equation*}
\text { Minimize } \quad \sum_{j \in N_{i}^{N}} \mathscr{4} \mathscr{H i}_{j i} X_{j i} \tag{8}
\end{equation*}
$$

such that:

$$
\begin{align*}
& \sum_{j \in N_{i}^{r}} p_{j} X_{j i} \geq \bar{p}  \tag{9}\\
& X_{j i}=0,1 \text { for all } j \in N_{i}^{r} \tag{10}
\end{align*}
$$

where all symbols are as defined earlier and $N_{i}^{r}$ denotes a sufficiently small neighborhood of $i$ with radius $r$, i.e. $N_{i}^{r}=\left\{j \in I: d_{j i} \leq r\right\}^{2}$. After solving the model described by [8-[10], which will be referred to as Model- B hereafter, we construct an association matrix $\mathbf{A}=\left[a_{i j}\right]$, where $a_{i j}=1$ indicates that unit $j$ is in the minimal compact district centered at unit $i$, i.e. if $X_{i j}=1$ in the optimal solution, and $a_{i j}=0$ otherwise.

We then select an optimal subset of all the compact districts generated as described above in such a way that: 1) each unit $j \in I$ is associated with at least $s$ of those minimal districts, where $s$ is an arbitrarily specified number; 2) the total number of associations across all $j \in I$ is maximized, and 3) the number of selected compact districts equals a desirably large number, denoted by $n$. The centers of the districts included in that optimal subset is then considered as $C$ in Model-A. The purpose here is to maximize the flexibility of unit assignment when determining the final district plan. The parameters $s$ and $n$ are specified arbitrarily by the user depending on the desired level of flexibility in unit assignment and the size of the district centers set $C$.

The problem described above is a mix of the set covering and maximal covering problems (Torregas and ReVelle, 1973; Church and ReVelle, 1974), which is given algebraically as follows (which we will call Model-C hereafter):

$$
\begin{equation*}
\text { Maximize } \quad \sum_{j} \sum_{i \in I} a_{j i} Z_{i} \tag{11}
\end{equation*}
$$

such that:

$$
\begin{align*}
& \sum_{i \in I} a_{j i} Z_{i} \geq s \text { for all } j  \tag{12}\\
& \sum_{i \in I} Z_{i}=n  \tag{13}\\
& \mathrm{Z}_{i}=0,1 \text { for all } \mathrm{i} \tag{14}
\end{align*}
$$

[^19]where $\mathrm{Z}_{i}=1$ means the minimal-compact district centered at unit $i$ is selected, thus unit $i$ is a potential district center to be used in Model-A, and $\mathrm{Z}_{i}=0$ otherwise, and $n$ is the number of potential district centers to be selected. Constraint [12] implies that each unit $j \in I$ must be included in at least $s$ of the minimal-compact districts to be selected, which allows the assignment of that unit to one of those centers in Model-A (thus allowing a minimal level of flexibility represented by $s$ possible assignments). Constraint [13] restricts the number of minimal-compact districts that must be selected among all the districts generated by Model-B. By maximizing the number of associations, represented by [11], the above model determines $n$ potential district centers spread optimally in space that will be used in Model-A for final selection to generate a politically unbiased district map ${ }^{3}$.

Contiguity is a difficult criterion to incorporate in a mathematical programming model particularly when working with a large number of spatial units to be districted. We accomplish this by requiring that if a spatial unit is assigned to a particular district center, then at least one of its neighbors (an immediately adjacent spatial unit) that is closer to the center must also be assigned to the same district. This is done by use of the following constraint:

$$
\begin{equation*}
X_{i j} \leq \sum_{\substack{k \in \mathrm{~N}_{j}, d_{i k}<d_{i j}}} X_{i k} \quad \text { for all } i \text { and } j \tag{15}
\end{equation*}
$$

where $\mathrm{N}_{j}$ is the set of immediate neighbors of unit $j$ (i.e. tracts that have a common edge with unit $j$-or adjacent to it). To see how this constraint works, suppose $X_{i j}=1$. Then the right hand side must be at least one, or for at least one of the variables in the summation we must have $X_{i k}=1$, which implies that one of the units adjacent to unit j whose distance to unit $i$ is less than the distance between unit $i$ and unit $j$. Applying the same argument to such a unit implies that there is a chain of mutually adjacent units that connect any unit $j$ to unit $i$ if the latter is a central unit of a district and unit $j$ is assigned to that district.

[^20]First we solve Model-B to generate tract-center associations for all units assuming that each unit can be a potential district center, and then solve Model-C (using the output of Model-B) to identify the optimal subset including a reasonably large number (specified exogenously) base units that will be considered as potential district centers in Model-A. A typical output of the first step is displayed in Figure 2a, where the yellow shaded area is formed by the tracts assigned to the central tract shaded in red. When solving Model-C, we assumed $s=3$, i.e. each tract must be associated with at least three potential district centers. Figure 2 b displays the centers optimally selected by Model-C.

It is unlikely that a given census tract can be part of a district centered at a remote unit, such as a unit in Chicago and a central unit near the southern tip of Illinois. For computational tractability, we rule out this possibility and for each potential district center $c \in C$ considered in Model-A we limit the set of units that can be assigned to $c$, say $I_{c}$, to an area that has two times the population of the average district population. This reduces the number of binary assignment variables in Model-A substantially. The units that will be actually assigned to $c$ (if included in the model solution as a district center) will be a much smaller subset of $I_{c}$, thus no loss of optimality occurs due to this artificial restriction. A typical case that occurred in the model solution is shown in Figure 3a, where the green-shaded area includes the tracts that could be assigned to the central unit shown in red, while the orange area displayed in Figure 3b includes the units that were actually assigned to that center.

When dividing each Senate district into two almost equally populated House districts, we first identify the four most northern, most eastern, most southern and most western tracts included in that Senate district ( 16 total) which are assumed as potential HRD district centers. The HRDs associated with the given Senate district are then created around two of those 16 potential HRD centers, which are determined endogenously, by using Model-A with $k=2$.


Figure 1: A graphical illustration of forming a compact district by minimizing the sum of (population weighted) distances between a central unit and all units assigned to the district. Each circle corresponds to the centroid of a base unit. R5 corresponds to the central unit of the district which includes four other closest and sufficiently populated units labeled as R1-R4.

Figure 2: Optimally selected candidate district centers. Magnified portion shows the optimally selected candidate centers in the Greater Chicago Area.



Figure 3a: A typical area (in green) around a potential center (in red) with a population size equal to twice the average district population (used to limit the possible unit-center associations)

Figure 3b: Tracts in Figure3a that are actually chosen by the model (in orange) to form a district around the central tract (in red).

## APPENDIX 2

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# How to Measure Legislative District Compactness If You Only Know it When You See it* 

Aaron Kaufman ${ }^{\dagger}$

Gary King ${ }^{\ddagger}$

Mayya Komisarchik ${ }^{\text {§ }}$
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#### Abstract

To deter gerrymandering, many state constitutions require legislative districts to be "compact." Yet, the law offers few precise definitions other than "you know it when you see it," which effectively implies a common understanding of the concept. In contrast, academics have shown that compactness has multiple dimensions and have generated many conflicting measures. We hypothesize that both are correct - that compactness is complex and multidimensional, but a common understanding exists across people. We develop a survey to elicit this understanding, with high reliability (in data where the standard paired comparisons approach fails). We create a statistical model that predicts, with high accuracy, solely from the geometric features of the district, compactness evaluations by judges and public officials responsible for redistricting, among others. We also offer compactness data from our validated measure for 17,896 state legislative and congressional districts, as well as software to compute this measure from any district. Word count: 9987


Replication Materials: Data, code, and other information needed to replicate all analyses in this article are available on the American Journal of Political Science Dataverse at Kaufman, King, and Komisarchik, 2020.

[^21]
## 1 Introduction

Compactness is treated in the law as an important legal bulwark against gerrymandering. The Apportionment Act of 1901, many court decisions, and 18 state constitutions require compactness for U.S. House districts, and 37 states require their legislative districts to be compact (see j.mp/aRED). Compactness is also required in federal law as one of the "traditional redistricting principles" which, when followed, can "defeat a claim that a district has been gerrymandered..." on the basis of race (Shaw v. Reno, 509 U.S. 630, 647, (1993)) or political party (Davis v. Bandemer, 478 U.S. 173, 2815, (1986)). ${ }^{1}$

Compactness is also important for the academic literature, where scholars seek to help the redistricting and litigation processes, and also to study venerable political science questions such as the causes, consequences, and normative implications of compact districts over American history (e.g., Ansolabehere and Palmer, 2016; Ansolabehere and Snyder Jr, 2012; Forgette and Platt, 2005). Compactness intuitively refers to both how close a legislative district's boundaries are to its geographic center and how "regular" in shape a district appears to be. But upon deeper study, scholars have shown that in fact compactness is a complicated multidimensional concept and have offered almost 100 measures of different features of it (e.g., Niemi, Grofman, Carlucci, and Hofeller, 1990). ${ }^{2}$

While many state constitutions explicitly require compactness, the vast majority provide no definition or measure for how to detect violations of the standard. For example, the Constitution of Illinois says only "Legislative Districts shall be compact". The Constitution of Hawaii requires that "Insofar as practicable, districts shall be compact." In Arizona, the Constitution orders that "Districts shall be geographically compact and con-

[^22]tiguous to the extent practicable." ${ }^{3}$
The federal courts have been similarly vague. They have acknowledged both the multitude of possible measures for compactness, and the fact that they often produce different conclusions. ${ }^{4}$ Except in rare cases, the courts have not provided guidance on particular measures or seen the need for them. For example, Justice Souter stated that "it is not necessary now to say exactly how a district court would balance a good showing on one of these indices against a poor showing on another, for that sort of detail is best worked out case by case" (Vieth v. Jubelirer, 541 U.S. 267 (2004); Souter dissenting). And most famously, a Supreme Court opinion indicated "One need not use Justice Stewart's classic definition of obscenity-'I know it when I see it'-as an ultimate standard for judging the constitutionality of a gerrymander to recognize that dramatically irregular shapes may have sufficient probative force to call for an explanation" (Karcher v. Daggett, 462 U.S. 725, 755 (1983)). Here, the Court at once laments the absence of a single quantitative standard while also implying that the concept is clear enough that all reasonable observers should understand it in the same objective way.

Consistently invoking the idea of "compactness" without a clear definition or required measure suggests two conclusions about the law. First, the law seems to imply that "compactness" is a single, coherent, and agreed upon concept, discernable simply by examining a district map. After all, how could the courts expect legislators to draw districts that comply with "compactness" without a shared understanding of what it means? And second, this lack of precision in the law has given redistricters and litigants battling over legislative maps in specific cases wide latitude to choose measures of compactness and reach

[^23]opposing conclusions (Defendant-Intervenors' Post-Trial Brief [at pp. 18], Bethune-Hill v. Va. State Bd. of Elections, 141 F. Supp. 3d 505 (E.D. Va. 2015) (No. 3:14 Civ. 852), ECF No. 104; and Motion In Limine Regarding Plaintiffs' New Compactness Test [at pp. 4], Vesilind v. Va. State Bd. of Elections, No. CL 15-3886 (Va. Cir. Ct. 3/31/2017).). Even when litigants might agree on the compactness of any one district by knowing it when they see it, systematically judging the compactness of many districts, or an entire redistricting plan, cannot be accomplished by merely looking. As such, the courts and policy makers tend to get very little benefit from the decades of work on quantitative measures of compactness offered by social scientists.

We attempt to span this divide between the seemingly universal understanding of compactness proposed in or needed for the application of the law, and the theoretical complexity and multidimensionality revealed in the social science literature. We do this by inferring, measuring, and validating the single underlying dimension of compactness that practitioners may need to apply the law, and we find that people of all types seem to agree upon it. In other words, since compactness in the law is, for all practical purposes, defined by the judgment of human observers - including redistricters, experts, consultants, lawyers, judges, public officials, and ordinary citizens - the claim of an objective standard, measured on a single dimension, can only be supported if most educated people evaluated a district's compactness in the same way. We provide this objective measure and show that these and other groups of observers all view compactness in accordance with it. This new dimension is not the average (or principal component) of existing measures but a new quantitative construction that accurately and reliably predicts human judgment.

In four sections, we proceed by conceptualizing, measuring, validating, and interpreting our derived dimension of compactness. Section 2 inductively defines the underlying dimension by building on the encyclopedia of existing diverse measures, adding new ones that show how humans perceive objects like district shapes, and providing intuition about the commonly perceived dimension we seek to measure. Section 3 then develops a way to measure this concept by eliciting views of the compactness of specific districts from respondents using a novel survey approach to rank order districts according to their com-
pactness. We are forced to develop a new method because the standard approach in the survey literature to a problem like this, Thurstone's paired comparisons, completely fails in our application. The high levels of intercoder and intracoder reliability produced by our alternative approach are consistent with a unidimensionality hypothesis (and suggests that our survey methodology may have other applications). This section then uses these results to build a statistical model that predicts with high accuracy how individuals rank districts, given only the the districts' shapes.

Our results enable us to apply one of the most important principles of statistics defining the quantity of interest separately from the measure used to estimate it - and, as a result, to provide evaluations that make our approach vulnerable to being proven wrong. We do this in Section 4 with cross-validation and then extensive out-of-sample validations in samples of public officials and judges from many jurisdictions, as well as redistricting consultants and expert witnesses, law professors, law students, graduate students, undergraduates, ordinary citizens, and Mechanical Turk workers. Application of this same principle also enables us to provide the first uncertainty estimates for a measure of compactness offered in the literature (see Supplementary Appendix D). Section 5 then offers interpretations of the resulting measure, and Section 6 concludes.

## 2 Conceptualizing

We now attempt to inductively characterize the concept of compactness that most laws, constitutions, judicial opinions, and participants in redistricting at least implicitly assume human observers intuitively understand.

As districting is "one area in which appearances do matter" (Shaw v. Reno, 509 U.S. $630,647,1993$ ), our approach is to measure the absolute compactness of the geometric shape of a district, separately from other facts that can impact this measurement such as geography or population. This is the most common basis for a compactness definition, dating well before the famous "Gerry-Mander" cartoon (Tisdale, 1812), but not the only one possible. Absolute compactness, in turn, may be constrained or influenced by fixed features of the state geography, such as rivers, coastlines, or highways. We measure the
quantity that would be influenced by these features, so that it measures the concept in the law and can be useful for further research. If a researcher had the alternative goal of defining and measuring relative compactness, based on how close it is to a realistic ideal, then our measure would be a key component in that calculation. In addition to measuring absolute compactness based on shape, our methods can also be used to measure compactness based on other criteria, such as population dispersion (Fryer Jr and Holden, 2011; Hofeller and Grofman, 1990; Niemi, Grofman, Carlucci, and Hofeller, 1990); see Section 3.3.

We attempt to characterize the compactness of each district separately. Although changing the boundaries of one district obviously affects neighboring districts, separate measurement follows major redistricting litigation, which typically evaluates the compactness of districts individually or in a small group rather than for an entire state redistricting plan all at once (e.g., Shaw v. Reno, 509 U.S. 630 (1993), pp. 637, 647, 656). This strategy is especially useful for the most fine grained scholarly research on the causes and consequences of compactness. ${ }^{5}$

Section 2.1 highlights empirical inconsistencies in existing shape-based measures to convey that the possible conceptual definitions of compactness, underlying these measures, are multidimensional. Then Section 2.2 provides intuition and tools to build toward a single concept of compactness.

### 2.1 Multiple Dimensions Underlying Existing Measures

Numerous specific compactness measures have been proposed in the academic literature, each one fitting different qualitative conceptual definitions and intuitions for certain ge-

[^24]ographical configurations and violating it for others (Altman, 1998; Niemi, Grofman, Carlucci, and Hofeller, 1990; Stoddart, 1965; Young, 1988). These measures are based on geometric concepts such as perimeters, areas, vertices, and centroids, often in comparison with some pure form geometric object such as a circle, rectangle, polygon, or convex hull. Each, however, focuses on a different dimension of what might be called compactness. Consider, for example, the five most frequently used measures by academic researchers, and also by experts in redistricting litigation: Length-Width Ratio, the ratio of the length to the width of the minimum bounding rectangle (Harris 1964; Timmerman, 100 N.Y.S. 57, 51 Misc. Rep. 192 (N.Y. Sup. 1906)); Convex Hull, the ratio of the area of the district to the area of the minimum bounding convex hull; Reock, the ratio of the area of the district to the area of a minimum bounding circle (Reock, 1961); Polsby-Popper, the ratio of the area of the district to the area of the circle with the same perimeter as the district (Polsby and Popper, 1991; Schwartzberg, 1965); and (modified) Boyce-Clark, the (normalized) mean absolute deviation in the radial lines from the centroid of the district to its vertices (Boyce and Clark, 1964; Kaiser, 1966; MacEachren, 1985). For details on these and others, see Supplementary Appendix A.

Without a gold standard, we cannot determine any measure's formal statistical properties, its error rates, or when it might fail. Although different measures are sometimes correlated, choices among these are presently made by qualitative judgment. Creative scholars have managed to use existing measures productively in research by combining multiple measures, adjusting or weighting each for specific purposes, or making careful qualitative decisions in specific cases (Ansolabehere and Palmer, 2016; Niemi, Grofman, Carlucci, and Hofeller, 1990).

We illustrate the issues with measuring compactness by presenting Figure 1, four state house districts from Alabama in 2000. Readers may wish to draw their own conclusions about the relative compactness of these districts, but we now provide in Table 1 an indication of how the most popular five measures rank them (we discuss X-Symmetry and significant corners in Section 2.2). As can be seen from the first five rows of Table 1, every one of these measures gives a different rank order for the four districts. We introduce
two new compactness measures in Section 2.2 for a different purpose; these are given at the bottom of Table 1 and also give unique rankings of the same districts. This example is merely a proof of concept, but finding such examples is easy: By random sampling, we estimate that in our collection of 17,896 state legislative and congressional districts (see Supplementary Appendix B), there exist 162 trillion sets of four districts such that every one of the seven measures provides a unique rank order. Of course, there is a large number from which to choose (this large number being about $0.15 \%$ of the total), but inconsistencies among in rankings on fewer than seven measures is both commonplace and is congruent with the long literature on this subject.
[Figure 1 about here.]
[Table 1 about here.]

### 2.2 Toward a Single Compactness Dimension

We now provide intuition helpful in turning the multiple types and dimensions of compactness illustrated in Section 2.1 into a single unidimensional concept underlying common conceptions, but in the absence of political or personal biases. We continue to proceed inductively, with Section 3 devoted to measuring this concept. We do this in three ways, followed by a characterization of the dimension of interest.

First, our goal is to elicit views about compactness, but without the biases psychologists have long demonstrated skew human judgments in the direction of our own political and other preferences. Although such unbiased views may be the goal of lawyers advocating on behalf of their clients, research has shown that subject matter experts are as vulnerable to bias as nonexperts, and more overconfident in the belief that they can avoid it. The only reliable solution has been to remove even the possibility of bias by instituting formal procedures (such as double blind experiments). (See Kahneman, 2011). We thus elicit views about compactness without revealing to respondents how their decisions in any one situation might benefit one political party or another. This is a critical point: Because individual judges, advocates, redistricters, and experts do not have access to the
mental processes in their own thinking that would enable them to evaluate and avoid these biases (Wilson and Brekke, 1994), they would also be unable to come to the same judgment as our measure in the context of a real redistricting contest by merely looking at a district shape.

Second, all existing compactness measures are rotationally invariant, meaning that if we rotate a district, say 45 degrees, a measure will have the same value. Although this is a reasonable normative standard from some perspectives - and we discuss below how to easily adjust our methods to impose this restriction if desired - human beings (including judges) do not evaluate districts in this way. In fact, human perception is famously sensitive to the rotation of objects: even familiar faces can become unrecognizable when viewed upside down (e.g., Maurer, Le Grand, and Mondloch, 2002). Our own experimentation done in R Shiny (Kaufman, 2020) suggests that people view long thin district shapes located on a diagonal ( N ) as less compact than the same shape located along the horizontal axis ( ). ${ }^{6}$ In contrast, legislative districts always have a well defined up (north) and down (south), as displayed on every commonly used map. Indeed, courts, redistricters, and judges virtually always use this single standard orientation and do not rotate districts when evaluating compactness; as a result, their decisions are not rotationally invariant. In other words, since the usual orientation of a district has precedence in how humans interpret it, some of our measures need to pick up on these features. ${ }^{7}$

Thus, primarily for illustration in this section, and later as a measurable feature of district shape that can be included (and if desired controlled) in our statistical model, we define here a new compactness measure that is not rotationally invariant. We do not intend this measure to substitute for other measures or to even be especially important on its own, but it will be useful to represent human perception. Thus, we define $X$-Symmetry by dividing the overlapping area, between a district and its reflection across the horizontal axis, by the area of the original district. Shapes like circles and rectangles have overlap regions equal to that of the original district and so have X-Symmetry values of 1. A long

[^25]thin district stretched out from top left to bottom right, or one like have X-Symmetry values close to zero. This measure, applied to the four districts in Figure 1, gives unique rankings for each; see the sixth row of Table 1.

Since we are attempting to quantify human perception, we try to avoid imposing theoretical notions of what compactness should be, what might be rational, or what meets various mathematically "pure" standards that implicate one normative preference or another (such as rotational invariance). Finding the common objective measure that exists in minds of districting authorities, the courts, and others requires respecting how humans think rather replacing it with alternative normative preferences. Although the courts have never addressed the issue, in all likelihood those who drafted compactness requirements in legislative statues, judicial opinions, and state constitutions, that imply that the concept is so simple that you know it when you see it, were not assuming rotational invariance. However, if a rotational invariant measure is desirable or at some point required, we can easily impose it using a procedure analogous to what we do for avoiding political bias. Thus, we would use all the procedures described in this paper except that we would simply display districts at random rotational angles when eliciting compactness evaluations.

Third, another feature of human perception is how we define what constitutes a "significant" feature of a district. If a roughly circular district has a ragged border, which of the small border inlets and peninsulas count as notable deviations from the circular shape? For example, suppose we give a large number of people the task of drawing from memory the shape of the continental United States. These drawings will all differ, but they will likely all include some of the same features - a roughly rectangular shape, a peninsula for Florida, a larger one for New England, and perhaps a somewhat rounded western ocean boarder. In other words, despite the enormous number of specific small features and vertices along the boarder to choose from, virtually all Americans are likely to recall, thus judging as significant, a small number of the same features.

To include this highly qualitative feature of human perception, we consider algorithms computer scientists design to list all of the "objects" in an image. There is no correct answer, but it turns out that different people are likely to give similar answers, and the
automation goal is to list the objects a human would identify. As we do with X-Symmetry, we illustrate this idea quantitatively, and give an example that will later become part of our model. To do this, we turn the geometric district shape into a set of pixels (i.e., changing from vector to raster representation), apply a corner detection algorithm (Shi and Tomasi, 1993), and count the number of "significant" corners. The more significant corners, the less compact the district by this metric. The last row of Table 1 gives the rankings of the four districts in Figure 1 according to the number of significant corners. This measure also gives the four districts a unique ordering.

Finally, we try to convey intuition about the underlying dimension of compactness we will quantify in the next section. We do this visually, by presenting in Figure 2 a set of districts that range from most (panel a) to least (panel d) compact. We find that almost anyone familiar with the district-based nature of modern democracy, and some sense of the word compactness, finds that district (a) is more compact than (b), which is more compact than (c), which is more compact than (d). The question is how to quantify this notion, so that it works for these four districts and all other geometric shapes, a topic to which we now turn.
[Figure 2 about here.]

## 3 Measuring

We now develop a more explicit measure of the concept of compactness to satisfy our requirements in Section 2. The immediate quantitative goal of the procedure is a continuous measure for each district, between 1 and 100, that estimates the expected rank a respondent would assign a district if embedded in a set with 99 others. With this measure, we can rank order any set of $n$ districts, given only quantitative measures of their geometric shapes.

To construct this measure, we first develop a method of eliciting views about compactness directly from survey respondents, something universally recognized as important but rarely done in this literature except informally by researchers (Angel and Parent, 2011; Chou, Kimbrough, Murphy, Sullivan-Fedock, and Woodard, 2014). Appendix A
attempts this by applying best current practices in survey research - using a modern version (David, 1988) of Thurstone's venerable paired comparisons (Thurstone, 1927), a method that dates at least to 1860 (Fechner, 1966). Under this approach, we pose a set of simple survey questions, each asking the respondent to decide which of two districts is more compact and, from the many answers, we construct the full ranking. We explain the motivation behind this approach and then demonstrate empirically that it utterly fails to accomplish its goal for this application.

Given the failure of paired comparisons, we have no choice but to develop a new approach. Thus, in Section 3.1, we turn to the method that paired comparisons was originally designed to supplant - asking respondents to rank many districts all at once. We show that, as we apply it, this approach turns out to work extremely well in our application (and may also work for many others too). As we describe, the supposed advantages of paired comparisons turn out to be disadvantages and the disadvantages of ranking turn out to be advantages. Section 3.2 takes the resulting survey elicitation method as our outcome variable, and new gold standard, and builds a statistical model to predict it from geometric features of the districts. Details about data used appear in Supplementary Appendix B.

### 3.1 How Ranking Outranks Paired Comparisons

Why does the method of paired comparisons perform so poorly? We propose four reasons, which together leads us to a workable approach for our application, full ranking - the method which paired comparisons originally supplanted.

First, although $\binom{n}{2}$ paired comparisons is vastly smaller than $n$ ! rankings (see the start of Appendix A), for some applications rankings make be quicker. After all, how long would it take to carefully and accurately rank 20 district shapes by their degree of compactness (or 20 friends by their heights or 20 animals by their friendliness)? A lot less than 2 quintillion seconds. What the idea behind paired comparisons seems to miss is that humans are excellent at pattern recognition and seeing the big picture. Humans also intuitively apply time-saving heuristics that reduce the complexity of tasks, such as in our application by grouping districts into distinct types, and considering all members of the group at once before analyzing members within the group.

Thus, in practice with full ranking, we have tried to ensure that respondents are using their big picture skills, such as by suggesting to them that they simplify the task by working hierarchically, first grouping districts into three coarse groups, and then producing groupings within each group, and finally starting from the top and checking and adjusting each district's position within the ranking; however, we found that heuristics and intuitions are strong enough that dropping these instructions did not degrade our full ranking approach. We also tried full ranking with districts printed on paper and arrayed on a long table, as well as via an online system we built that allows districts to be dragged and dropped to their chosen location; we find no evidence that the mode of administration matters either (as with Blasius, 2012).

Second, human respondents work better when motivated and engaged. While paired comparisons successfully avoid the risk of asking respondents questions they do not understand, it is also an unavoidably boring and tedious task, especially after the first few questions. In contrast, ranking a large set of districts is more intellectually challenging and engaging (Fabbris, 2013). Our own cognitive debriefing strongly supports the advantages of ranking in this regard. ${ }^{8}$

Third, if it is possible for a survey respondent to rank (say) 20 districts without much trouble, then we can save considerable time by administering this one engaging survey task rather than having to ask 190 tedious paired comparisons for each respondent. Ranking would then save considerable time, expense, and respondent fatigue (Ip, Kwan, and Chiu, 2007). As a hint that this might work, Krosnick (1999) (studying rating rather than paired comparisons) finds that often "rankings give higher quality data than ratings".

And finally, the literature makes clear that compactness is a multidimensional concept (Niemi, Grofman, Carlucci, and Hofeller, 1990). Yet, we are trying to tap into a single unidimensional concept of compactness that we hypothesize respondents, if given the choice, would select and use. In this light, the fact that Thurstone's approach enables respondents to make each paired comparison independently of the others allows, and may

[^26]even encourage, them to use different dimensions for different comparisons. In other words, while "roundness" may be the deciding factor for compactness in one given pair of districts, length vs. width may be the relevant question in the next pair, and so forth. This may then be what results in the low levels of intercoder and intracoder reliability we have documented. In contrast, ranking has the advantage of encouraging respondents to choose a single dimension of compactness and to use it for all their decisions. With paired comparisons, the only way to do this would be to ask respondents to choose a single dimension explicitly and to keep that dimension in their heads while they answer 190 randomly ordered survey questions. Although the goal of any survey question is to be clear enough so respondents are answering the question intended by the researcher (i.e., on the dimension of interest), giving respondents multiple separate questions makes this difficult to achieve.

To test our hypothesis that ranking will work better than paired comparisons, we ask respondents to give a full rank order for 100 separate legislative districts by their degree of compactness.

To begin, we embed our 40 districts (which we used in 20 pairs in the experiments in Figures 7 and 8) among 60 others and ask a new set of respondents to rank all 100. To compute a relative assessment of the two methods, we evaluated intercoder and intracoder reliability of the implied paired comparisons of how these 20 pairs were ordered by full ranking and compared them to reliability from the actual paired comparisons. That is, from full ranking, we record only which district in each pair of 20 comparisons is ranked higher. Then, to compute intracoder reliability, we waited two weeks, shuffled the rank ordering, and asked the same respondents to rank the same 100 districts, again only using the 20 designated pairs among these. We then computed the percent agreement over time in these implied paired comparisons exactly as we did for the actual paired comparisons. The results, which appear in the same two figures (salmon colored histogram, at the right of each figure), are far more clearly separated from the random placebo test and have much higher levels of intracoder reliability than the actual paired comparisons. For intercoder reliability, in Figure 7, we have 75\% agreement on average, and for intracoder reliability,
in Figure 8, we have 88\% agreement on average.
Now that we have a method that bests paired comparisons for measuring compactness with respect to pairwise intracoder and intercoder reliability, we turn to evaluating full ranking on its own terms. We begin with intercoder reliability by correlating the ranks for 100 districts coded independently by (all possible) pairs of respondents. We then present in Figure 3 one scatterplot representing the pair of coders with the median correlation ( $\rho=0.77$ in the top left panel) as well as the pair with a correlation in the first quartile (bottom left) and in the third quartile (top right). In the bottom right of the same figure (salmon colored), we also present a density estimate (using a kernel truncated at the minimum and maximum observed correlations) of all the correlations, along with a baseline density estimate of correlations among randomly generated ranks. The conclusion from this figure reveals high intercoder reliability, clearly distinguishable from chance, and with no systematic error patterns in any individual scatterplot.
[Figure 3 about here.]

We then repeat this process for intracoder reliability by correlating the ranks for each respondent with the same respondent, re-ranking the same districts, two weeks later. Figure 4 shows these results in the same format as Figure 3. As would be expected, our results here are even stronger than for intercoder reliability. The median correlation (top left) is $\rho=0.9$, with not much spread around the median (see salmon colored histogram in the bottom right panel). None of the scatterplots show any systematic patterns in deviations from the $45^{\circ}$ line, and all indicate high levels of intracoder reliability.
[Figure 4 about here.]

### 3.2 A Statistical Measurement Model

To construct our ultimate measure of compactness, we begin with a set of districts and elicit the views of respondents via our full ranking survey approach. In Section 3.1, we describe this survey methodology. Supplementary Appendix B gives details of how we recruited our survey respondents, collected our set of districts, conducted our experiments,
and wrote and presented the ranking task to respondents. We also discuss there the mechanism for how we elicited ranking preferences, both in person (sorting paper cards with districts printed) and online (dragging and dropping district images).

Our data collection process results in six sets of 100 districts, each ranked by several individuals or pairs of individuals working independently. We average away random error by calculating the first principal component of the rankings produced for each set of 100 districts, preserving the ranked scale. This first principle component, a summary of human-derived compactness rankings, forms the outcome variable in our statistical model, using only information from the shape of districts as predictors. To produce our predictor variables, we calculate a set of geometric features including all seven compactness indicators from Table 1 and the others described mathematically in Supplementary Appendix A.

Finally, we train an ensemble of predictive methods with these data, consisting of least squares, AdaBoosted decision trees, support vector machines, and random forests. Supplementary Appendix C gives the details of these methods and of how we construct this ensemble and its component parts.

All further details and code are available in our replication data file which accompanies this paper. In the same way that logit or ordered probit take discrete outcome variables and generates continuous predictors, our training data consists of integers from 1 to 100 , but our ensemble model produces continuous outputs.

### 3.3 Compactness as Shape and Population Dispersion

As described in Section 2, the concept of compactness in the law, most of the literature, and our paper is based on district shape alone. However, other conceptualizations may be of interest for some purposes, such as based on population, communities of interest, not dividing political subdivisions, etc. For each of these, all the methodological procedures we developed in this paper should be directly applicable. The measure that results from the application of our procedures entirely depends, of course, on the quantity of interest being estimated, and there is no guarantee that a measure of compactness based on shape will be related to one based on other criteria.

As one small proof-of-concept of the applicability of our approach, we repeated our survey with district shapes that also represented where in each district people live (Ansolabehere and Palmer, 2016; Niemi, Grofman, Carlucci, and Hofeller, 1990). We ran this population distribution experiment with six undergraduates from different universities on the same set of 20 districts. Details of the experimental protocol appear in our replication data set. Results indicate that the median correlation between the $\binom{6}{2}=15$ possible pairs of rankings was a substantial 0.94 , with a range of 0.12 . This is comparable to the results we found using shape alone.

## 4 Validating

Via cross-validation (in Section 4.1) and out-of-sample prediction in diverse populations (in Section 4.2), we now evaluate our single, unidimensional compactness measure, deterministically computed from a district shape, and confirm our hypothesis that the theoretical concept we are measuring is the same one people know when they see. The data for this section come from diverse populations including participants directly involved in decision making about legislative redistricting.

### 4.1 Cross-validation

We evaluate our model here with cross-validation, where each fold reserves one of our six sets of 100 districts. To do this, we use six groups of survey respondents, potentially making it harder for our model by mixing size of group, mode of administration, and type of respondent: (1) two pairs of undergraduates (the two within each pair working together) and one pair of graduate students; (2) one pair of undergraduates, one individual undergraduate, and one pair of graduate students; (3) 5 individual undergraduates, 5 pairs of undergraduates, and 16 Mechanical Turk workers; (4) 5 pairs and five individual undergraduates; (5) 8 undergraduates; (6) 8 undergraduates. (We found ex post that respondents gave similar rankings regardless of whether they worked alone or in pairs. Similarly, Mechanical Turk workers, undergraduates, and graduate students gave similar rankings on the same sets of districts.)

We then trained our model on groups 1-5 of respondents taken together, and predicted the remaining "test set" of respondents in group 6; we repeated this six times in total, with each group taking its turn as the test set and the remaining groups as the training set. The prediction from this model uses all information from the training set but only the district geometry (i.e., no survey information) from the test set. Figure 5 evaluates the performance of this procedure by providing six scatterplots corresponding to each of our training set-based predictions (horizontally) by the true test set values (vertically). As is evident, these cross-validation results indicate very high predictive accuracy. Correlations between predictions and test set values range from 0.92 to 0.96 , with no noticeable systematic error patterns in any graph.
[Figure 5 about here.]

### 4.2 Predictive Validation in Diverse Populations

The statistical model in Section 3.2 is designed to predict human judgment about the compactness of any set of districts, given only the geometric shapes of the districts. Our model can make a prediction for any legislative district shape, including new districts and those that do not appear in our training set.

Our hypothesis is that any informed human being will judge the compactness of a set of districts in almost the same way, thus admitting to high levels of statistical reliability. We now test this hypothesis by asking a wide range of groups to evaluate the compactness of different sets of legislative districts and comparing these evaluations to our predictions. Our main test comes from 96 sitting justices, judges, and public officials, all with some responsibility for redistricting or deciding redistricting cases. We also elicited the views of 102 others, ranging from less to more involved in and knowledgeable about redistricting, including Mechanical Turk workers, who received small monetary payments, undergraduates, some of whom received hourly wages, and others who were not paid, including political science PhD students, law students, law faculty, redistricting consultants and expert witnesses, and lawyers involved in legislative redistricting cases.

We promised our respondents confidentiality, including their responses and the fact of
their participation. This was most obviously a concern in recruiting judges and justices, who decide redistricting cases, and other public officials, who have decision making authority in or substantial influence on the process. It turned out to be of no less a concern for some lawyers who try redistricting cases, and some consultants and expert witnesses who are held to account for their previous statements and opinions. For these reasons, we are not able to make these data available publicly, although we do make available the software we designed to let respondents sort districts online and all our specific experimental protocols. All these steps were approved by our university Institutional Review Board. (We have also prepared and field tested teaching exercises for American government classes that use our districts, enable students do the ranking exercise themselves, and compare them to our predictions.)

In this experiment, we asked each respondent to rank order twenty legislative districts, not included in our training data, by their degree of compactness; we represent the degree of predictive accuracy by a simple correlation with our predictions. All respondents ranked the same twenty districts. We portray our results in Figure 6 with a histogram for each of nine categories of people. As a baseline, we present a density estimate (in blue) of the percent agreement among random rankings, which is of course centered at zero, and the variance of which conveys uncertainty given $n=20$ districts. The (salmon-colored) histogram is for Mechanical Turk workers. The remaining histograms of correlations appear in white, with black outlines. We do not distinguish among these for a further level of confidentiality, but they all lead to the same conclusion of very high levels of predictive accuracy.

## [Figure 6 about here.]

We found no statistically significant differences between the size of the correlations among different groups of respondents. The main predictor of the strength of the correlations was the time spent on the task, with longer times yielding higher correlations. This accounts for the larger variance of Mechanical Turk workers, as they are paid by the completed task regardless of how long they spend.

## 5 Interpreting

Having conceptualized, measured, and validated our estimate of compactness, we now interpret the result. Of course, we already have one interpretation - that we know it when we see it. That is, our fully automated quantification of the compactness of a district geography reproduces how informed human observers evaluate a never-before-seen district shape. Our model can do this quickly for millions of potential districts in ways no human could ever do - and so it could be used in a court case comparing entire legislative plans or in academic research comparing many legislatures - but the quantity being estimated by our model and by individual people is the same.

Nevertheless, a reasonable question is whether we can understand compactness via some simpler geometric approach, analogous to any of the existing measures. The common difficulty of explaining how we as humans (or statistical models that approximate them) perform sophisticated tasks - recognizing a friend's face, developing a scientific hypothesis, judging compactness when we see it, etc. - is known as "Polanyi's paradox," that "we know more than we can tell" (Autor, 2015; Polanyi, 1966). We have studied, in considerable detail, how to simplify our measure and find that indeed the simplest way to know what we see is merely to look or to use our measure. A theoretically simpler version may even be an illusory goal, since humans use such sophisticated combinations of these mathematical simplifications rather than any one. We analyze this point in four ways, and then discuss whether other approaches to this question might be possible.

First, we offer a direct answer from our extensive qualitative analyses of the outputs of our approach along with the features that are most predictive: Our measure of compactness favors districts that are squarish, with minimal arms, pockets, islands, or jagged edges. (We use "squarish" rather than "circle-like" because many real districts are approximately square-shaped but almost none resemble circles.) Importantly, no existing compactness measure estimates a theoretical quantity that can reasonably be described in this way.

Second, we offer illustrations of the nature of the agreements and disagreements between our measure and each of the seven existing measures we discussed in Section 2.

For each existing measure, we construct a $2 \times 2$ cross-tabulation of example districts that reflect agreements (compact and noncompact) and disagreements (where the existing measure says noncompact and ours compact, and the reverse). We array horizontally the four cells of this $2 \times 2$ table for each measure in a row in Table 2. To generate this table, we define "compact" districts as having a predicted compactness rank in the top 15 (of 100) and "noncompact" as 85 or lower. (If no district appears in a cell of the cross-tabulation, we expand our definition from 15 and 85 to 20 and 80 , etc.) Then, to avoid cherry picking, we choose the first in alphabetical order among all districts defined by each cell in each table. ${ }^{9}$
[Table 2 about here.]

The results in Table 2 are striking. The agreements appear in the first two columns: Column one includes seven obviously compact districts, and column two includes seven clearly noncompact districts. The last two columns reflect disagreements between our measure and an existing one. The first of these (in the third column) are districts that our measure indicates are noncompact and an existing measure says are compact. Most human observers agree with our measure (by design) that these are in fact highly noncompact districts. Similarly, the final column includes districts judged as noncompact by an existing measure, but compact by ours. This table clearly reveals how each existing measure picks up important features of the compactness of legislative districts and omits others. The features each measure picks up or misses are those widely discussed in the existing compactness literature as benefits or failures of each measure, since in practice this theoretical literature is using the standard from which our measure was derived (you know it when you see it) to judge their own measures. In contrast, our measure seems to pick up all the features identified throughout the literature as desirable, without obviously missing any feature of a district shape generally seen as important.

Third, do different measures generate different conclusions in practice? The answer here depends on in which legislature the comparison is being made. For any two measures,

[^27]it is easy to draw a districting plan where the measures change the rankings of compactness in any arbitrary way. We could also be misled by stacking up data across legislatures - and thus ignoring the bias from heterogeneous treatment effects - in which case we would see that our measure correlates quite low with most measures but at about 0.9 for convex hull and Polsby-Popper, and similarly high correlations for the naive average of all measures. In fact, the only coherent way to answer the question is to use real world legislatures, which is the context in which comparisons matter and, as it turns out, where differences are significant. To pick an extreme case from the current US Congressional map, Polsby-Popper correlates with our measure (i.e., the measure any human observer would choose when evaluating districts) at 0.95 in Indiana's 1970 map but -0.37 in its 1890 map. We thus study this question more systematically by analyzing the 778 legislatures from our data with unique sets of district boundaries (i.e., for every available state, legislative chamber, and year; e.g., Alabama State Senate in 1962). Comparisons across measures in court mostly depend on which district or plan ranks highest and so we compute the percent of times, across data sets, where each existing measure has the highest correlation with our measure. The measure that winds up in the top position most often is Convex Hull, but this occurs in only $54.5 \%$ of the data sets - followed by the Polsby-Popper in $31.0 \%$, Grofman in $6.2 \%$, Y-axis Symmetry in $1.9 \%$, Reock and X-axis Symmetry at $1.6 \%$ and $1.5 \%$, and Boyce-Clark at $0.6 \%$; even measures such as the area of the minimum bounding circle and the number of discontiguous polygons correlate most highly sometimes. In other words, any existing measure can come out on top in approximating our measure depending on the particular features of the set of district shapes that make up the legislature, and so none of these measures alone can be used as a simpler replacement with our measure of what people know when they see, without checking the relationship first (see Supplementary Appendix E).

Finally, the best practice in choosing predictive models, which we followed, involves finding the most parsimonious model that predicts accurately; as such, we are by definition unable to find an even more parsimonious model without giving up predictive accuracy. Thus, we searched for a more parsimonious model that degraded performance by only
a small amount. Unfortunately, we found no large discontinuity in the relationship between parsimony and performance. A straightforward principal component analysis of the existing measures also does not yield a simple solution.

In summary, this section demonstrates that none of the existing measures, and no measure we could find, offer a simple geometric representation for what humans know when they see. To be clear, however, we have not proved that creating such a measure is impossible. We thus leave this as an open question and encourage future researchers to seek such a simplifying geometric definition, if that turns out to be possible.

## 6 Concluding Remarks

We conclude that the measure derived here reflects the underlying viewpoint held about the concept of compactness by everyone from educated Americans to public officials, judges, and justices. This measure appears to confirm and reflect the single, universally recognizable standard implicit in legal compactness requirements of state constitutions, federal and state legislation, and court decisions. Although "we know more than we can tell" about how humans perceive compactness, this measure quantifies "what we know when we see." The measure is also visibly different (as per Table 2) from any existing measure and, by design, much closer to how human beings perceive compactness.

Approaches developed here for measuring an ill-defined concept that you know only when you see may also be applicable to other difficult-to-define concepts. These include measurement by full ranking rather than paired comparisons, which saves time and turns out, in our application, to have much higher levels of intra- and intercoder reliability; the incorporation in a model rather than replacement of most existing measures and approaches; and formalization into a statistical model of an approach that predicts the views of a wide range of different types of people.

The key aspect of our approach here is defining the concept of interest separately from the measure used to estimate it, so that our measure becomes vulnerable to being proven wrong and, as a result, our approach can improve over time. In this light, we encourage others to take up this challenge and improve on the methods we propose, and develop
statistical methods that outperform ours; this may now be possible, as clear performance standards now exist. New features measuring compactness can also be included in our approach as additional covariates in our statistical model, which may well be improved.

We hope the large collection of compactness data we make available with this paper (for 17,896 state legislative and congressional districts) and software that makes it easy to compute compactness on any new district enable future researchers to study a wide range of questions related to this crucial concept (see Appendix E). As well, we hope that having a single measure of compactness that all agree on will begin to constrain some aspects of unbridled advocacy during the redistricting process and subsequent litigation.

## Appendix A How Paired Comparisons Fails

The method of paired comparisons has been touted for more than a century and a half for its two key advantages. First, this approach puts fewer demands on survey respondents than asking respondents to do a full ranking. That is, to produce a ranking of $n$ items requires the choice among $n$ ! possible rankings, whereas the same information can be elicited with only $\binom{n}{2}$ paired comparisons. This is not trivial since $n!\gg\binom{n}{2}$; for example, with $n=20$, we have $20!=2.4 \times 10^{18}$, or 2 quintillion possible rankings, whereas $\binom{20}{2}=190$ paired comparisons is large but still manageable in a single survey (and may even be reduced; see Mitliagkas, Gopalan, Caramanis, and Vishwanath 2011). For these reasons, Converse and Presser (1986, p.28) comment on a historical example with only 13 items: "Tasks of this scope were soon seen as much too difficult..., and in our own time, rank orders of this size are all but invisible in the literature". Thus, if full ranking is used, the best practice has been "not to use lists longer than three or four items" (Gideon, 2012).

Second, Thurstone's approach only requires simple questions that are easy to understand, concrete, and specific. With it, we ask a respondent which among a pair of legislative districts is more compact, and then repeat this simple question multiple times with different pairs of districts. Then, after eliciting information in this manner, the researchers combine these binary decisions into a ranked scale (using Guttman scaling or a more so-
phisticated approach accounting for measurement error; e.g., Mitliagkas, Gopalan, Caramanis, and Vishwanath 2011). The method assumes all respondents will use the same unidimensional scale to make their choices for all their paired comparisons (an issue we return to). The supposed advantage of this approach is that respondents are asked only what they know (a paired comparison) and researchers do what they are better at, which is taking on the complicated task of inferring the underlying full ranking from all the elicited information.

To apply this method, we conducted multiple iterated rounds of pre-testing and cognitive debriefing while adjusting question wording and how the districts appeared ${ }^{10}$. But despite dozens of trials over many months, testing numerous variations, and with a wide range of research subjects, online and in person, our inter- and intracoder reliability statistics were rarely much above random chance. To see what we found, consider a simple experiment with 40 respondents (in this case on Amazon's Mechanical Turk), each asked to choose the more compact district from each of twenty pairs, producing a 20-length binary decision vector. This survey enabled us to compare the percent agreement among the 20 decisions for each of $\binom{40}{2}=780$ pairs of respondents. Figure 7 gives a histogram of these percent agreements (in blue, marked "paired", computed as a density estimate). For comparison, we also generate a placebo test, under the null hypothesis of no agreement, by randomly generating 780 pairs of 20-length vectors and computing from them the percent agreement and plotting its histogram (white with a black outline, marked "Random"). (We discuss the "Ranking" figure in the next section.)
[Figure 7 about here.]
As expected when comparing coin flips, the random placebo percent agreement is centered at $50 \%$. In contrast, the paired comparison percent agreement histogram is shifted farther to the right than the placebo histogram, but the mean only moves to $54 \%$, leaving the two distributions with considerable overlap. Put differently, the best we could do with the method of paired comparisons, even before the step of turning paired decisions into rank orders, is results with unacceptably low levels of intercoder reliability.

[^28]We now rule out the possibility that these results are due to different people having incompatible notions of compactness by studying intracoder reliability. To do this, we waited two weeks, randomly shuffled the order of the 20 paired comparison questions, and administered the survey to the same people. (Of the 40 people, only one mentioned, on post-survey cognitive debriefing, that "some" of the districts may have been the same as the first week.)

These results appear in Figure 8 (also as a blue histogram marked "Paired") and are more distinct from the random placebo test (in white with a black outline marked "Random") than with intercoder reliability in Figure 7, as would be expected. The mean of the paired comparison histogram is now at $65 \%$ agreement, although the overlap with the random distribution is still large. (We discuss the third histogram in the next section.)

## [Figure 8 about here.]

We thus conclude that these standard, best practice approaches are inadequate, at least for our application, and turn to an alternative. See Section 3.

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Figure 1: Four Districts from the Alabama State House in 2000.


Figure 2: The Underlying Compactness Dimension, from most compact (a) to least compact (d) (all five of the most common compactness measures agree with this ordering). (Districts include, (a) Wyoming State House District 42, 2010; (b) Pennsylvania State House District 185, 2010; (c) Oklahoma Congressional District 1, 1950; (d) Louisiana State Senate District 3, 2010.)


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## Legislative Districts

|  | AL 21 | AL 9 | AL 62 | AL 1 |
| :--- | :---: | :---: | :---: | :---: |
| Convex Hull | 1 | 2 | 3 | 4 |
| Reock | 4 | 3 | 2 | 1 |
| Polsby-Popper | 2 | 3 | 1 | 4 |
| Boyce-Clark | 3 | 4 | 1 | 2 |
| Length/Width | 4 | 2 | 3 | 1 |
| X-axis Symmetry | 4 | 1 | 2 | 3 |
| Significant Corners | 3 | 1 | 2 | 4 |

Table 1: Seven Unique Compactness Rankings of the Same Four Districts: Five Existing and Two New Metrics


Table 2: Illustrations of agreements (in the first two columns) and disagreements (in the last two columns) about the degree of compactness between each of seven existing measures and our measure. Each row represents a $2 \times 2$ table of our measure by an existing measure, with a dichotomized compactness summary, displaying one example district in each cell arbitrarily chosen via alphabetical order.


[^0]:    ${ }^{1}$ In both cases, courts made small changes to the maps created by the unbalanced commissions, but these did not fundamentally alter their partisan character.
    ${ }^{2}$ The data points in Figure 1 are proportional in size to the 1990 vote totals, and it is clear that the smallest seats were also the least Republican. The requirement that new districts have nearly equal populations means that vote totals vary less across district in 1992 than they did in 1990. In turn, the figures feature many more points above the 45 -degree line (i.e. increased Republican safety) than below because the final election before a census usually sees GOP wins in over-sized seats and Democratic wins in under-sized seats. ${ }^{3}$ Puppe, Clemens and Attila Tasnadi. 2008. "A computational approach to unbiased districting." Mathematical and Computer Modeling 48, 9-10 (November): 1455-60.

[^1]:    ${ }^{1}$ Interviews were conducted online, between April 30 and May 10, 2010. Respondents were located by sample matching: a random sample was drawn from the full Illinois voters list, then each name was matched to a member of the YouGovPolimetrix panel by demographic and other variables.

[^2]:    ${ }^{2}$ In another rare example of survey data on redistricting, Tate reports that most black survey respondents do not like racial gerrymandering when initially presented with the idea, but large number can be persuaded to support it when they are presented with arguments in its favor. See Tate, Katherine. 2003. "Black Opinion on the Legitimacy of Racial Redistricting and MajorityMinority Districts." American Political Science Review 97, 1 (February): 45-56.

[^3]:    ${ }^{3}$ See Gaines, Brian J. and James H. Kuklinski. 2010. "To Gerrymander or Not: What kind of Electoral Districts does the Public Want?" Illinois Issues 36, 9 (September): 30-33.

[^4]:    ${ }^{1}$ As will be discussed later in this chapter, gerrymandering may also be driven by minority representation requirement. Congressional District \#4 in Illinois is a typical example of this.

[^5]:    ${ }^{2}$ A comprehensive review of the related literature can be found in Williams (1995).
    ${ }^{3}$ For instance, Hess et al. (1965) report that their computer model generated contiguous and much more compact legislative districts plans than the existing district plans of Delaware. Furthermore, their plans reduced the difference between maximum and minimum district populations from about 6.3 thousand to 1.8 thousand for Senate districts and from 4.3 thousand to 1.2 thousand for House of Representatives districts.

[^6]:    ${ }^{4}$ These are specific to the Illinois legislative districting case. With proper modifications the modeling methods used here can be applied to any redistricting problem. The number of spatial units and districts to configure should be specified accordingly.

[^7]:    ${ }^{1}$ Those states with only one member of the U.S. House of Representatives do not draw legislative maps since their respective representatives run at large in the state.

[^8]:    ${ }^{2}$ The bill/public law that holds the definition of Illinois' congressional district borders for the 2000s is HB 2917/P.A. 920004, which passed in 2001. The Illinois Redistricting Commission filed its plan for the state legislative maps with the Secretary of State in October.
    ${ }^{3}$ The exceptions here are those relatively few state legislative districts that have more than one senator or representative for them-so called, "multi-member districts."

[^9]:    ${ }^{4}$ Richard K. Scher, Jon L. Mills, and John J. Hotaling, Voting Rights and Democracy (Chicago: Nelson-Hall, 1997).
    ${ }^{5}$ C. Herman Pritchett, "Equal Protection and Urban Majority," American Political Science Review 58(1964):869-875.
    ${ }^{6}$ The pivotal cases on state legislative redistricting in this period were Baker v. Carr, 369 U.S. 186 (1962); Reynolds v. Sims, 377 U.S. 533 (1964); and Lucas v. 44th General Assembly of Colorado, 377 U.S. 713 (1964); a key case of this period regarding congressional districts was Wesberry v. Sanders, 376 U.S. 1 (1964).
    ${ }^{7}$ The principle was originally referred to as "one man, one vote."

[^10]:    ${ }^{8}$ James L. McDowell, "The Orange-Ballot Election: The 1964 Illinois At-Large Vote and After," Journal of Illinois History 10 (2007): 289-314.

[^11]:    ${ }^{10}$ Richard Forgette, Andrew Garner, and John Winkle, "Do Redistricting Principles and Practices Affect U.S. State Legislative Electoral Competition?" State Politics and Policy Quarterly 9(2009):151-175; Jason Barabas and Jennifer Jerit, "Redistricting Principles and Racial Representation," State Politics and Policy Quarterly 4 (2004): 415-436.
    ${ }^{11}$ Karcher v. Daggett, 462 U.S. 725, 730-31 (1983); White v. Regester. 412 U.S. 755 (1973).

[^12]:    ${ }^{12}$ Hunt v. Cromartie 526 U.S. 541 (1999); Shaw v. Hunt 517 U.S. 899 (1996); Miller v. Johnson 515 U.S. 900 (1995).
    ${ }^{13}$ Easley v. Cromartie 532 U.S. 234 (2001).

[^13]:    ${ }^{14}$ Shaw v. Reno 509 U.S. 630 (1993).
    ${ }^{15}$ Barabas and Jerit, op cit.; Forgette, Garner, and Winkle, op cit.; Jonathon Winburn, The Realities of Redistricting: Following the Rules and Limiting Gerrymandering in State Legislative Redistricting (Lanham, MD: Lexington, 2008); David Butler and Bruce Cain, Congressional Redistricting: Comparative and Theoretical Perspectives (New York: Macmillan, 1992).

[^14]:    ${ }^{16}$ Shaw v. Reno; National Conference of State Legislatures, Redistricting Law 2010 (Denver, CO: National Conference of State Legislatures, 2009).
    ${ }^{17}$ Shaw v. Reno; Abrams v. Johnson 521 U.S. 74 (1997). The numbers of states noted here are those that use these criteria for state legislative and/or congressional redistricting.
    ${ }^{18}$ Shaw v. Reno; Bush v. Vera 517 U.S. 952 (1996); DeWitt v. Wilson 856 F. Supp 1409 (E.D. Cal 1994).
    ${ }^{19}$ Shaw v. Hunt 517 U.S. 899 (1996).
    ${ }^{20}$ Miller v. Johnson; Abrams v. Johnson.
    ${ }^{21}$ Tarry Hum, Redistricting and the New Demographics: Defining
    "Communities of Interest" in New York City. (New York: City University of New York, 2002)
    ${ }^{22}$ Johnson v. Abrams.
    ${ }^{23}$ Johnson v. Abrams.

[^15]:    ${ }^{24}$ Unless otherwise noted, the discussion this section and the data in Table 2 apply strictly to state legislative redistricting. All but six states draft and adopt the maps for their U.S. House districts with the same mechanism as for state legislative redistricting. Six state legislative commission states (Alaska, California, Colorado, Missouri, New York, and Pennsylvania) recognize that state legislatures have less direct conflict of interest in drawing congressional districts than their own, so they carry out congressional redistricting through their regular legislative process.

[^16]:    ${ }^{25}$ Michael P. McDonald, "A Comparative Analysis of Redistricting Institutions in the United States, 2001-02," State Politics and Policy Quarterly 4(2004):371-95.

[^17]:    ${ }^{26}$ Douglas Johnson, Ian Johnson, and David Meyer, "Redistricting in America: A State-By-State Analysis." Report of the Rose Institute of State and Local Government, Claremont, CA (2010).
    ${ }^{27}$ It is also important to note that if a state's process does not yield a set of maps before the next election, state or federal courts may step in to draw them. But since the 1960s, this has been uncommon.

[^18]:    ${ }^{1}$ The distances can be the shortest path distances from centroid to centroid or edge-to-edge. Here we use the latter.

[^19]:    ${ }^{2}$ Since $i$ is fixed, typically this problem is easy to solve even if we consider all $j \in I$, but restricting to a small neighborhood saves processing time and avoids potential computational difficulties.

[^20]:    ${ }^{3}$ A simple analogy to this is the following: Suppose each unit corresponds to a parking space in a parking lot to be lighted by $n$ light poles we wish to place. If a parking space receives sufficient lumens from a pole we assume that it is lighted by that pole, otherwise it is not. This is represented by the association matrix defined above. For safety, we require each parking place to be lighted by at least $s$ poles. The best placement of the light poles would be the one that maximizes the amount of lighting (lumens) provided to all parking spaces in the entire lot while at the same meeting the minimum lighting requirement for each spot. The best lighting strategy is analogous to the most flexible center selection strategy where we wish to maximize the choices when assigning individual tracts to selected centers.

[^21]:    *Winner of the 2018 Robert H. Durr Award from the Midwest Political Science Association. Our thanks to Steve Ansolabehere, Fred Boehmke, Ryan Enos, Dan Gilbert, Jim Griener, Bernie Grofman, Andrew Ho, Dan Ho, James Honaker, Justin Levitt, Luke Miratrix, Max Palmer, Stephen Pettigrew, Jamie Saxon, Steve Shavell, Anton Strezhnev, Wendy Tam, Rocio Titiunik, Larry Tribe, Robert Ward, participants in "A Causal Lab", and the audiences at the Society for Political Methodology Meetings, Nuffield College at Oxford University, the Harvard Applied Statistics Workshop, and the ICPSR Summer Program for helpful data or suggestions; and to Stacy Bogan, the Center for Geographic Analysis, and the Institute for Quantitative Social Science at Harvard University for research assistance and support.
    ${ }^{\dagger}$ Assistant Professor, New York University Abu Dhabi, Saadiyat Island, Abu Dhabi, UAE; AaronrKaufman.com; aaronkaufman@nyu.edu, (818) 263-5583.
    ${ }^{\ddagger}$ Albert J. Weatherhead III University Professor, Institute for Quantitative Social Science, 1737 Cambridge Street, Harvard University, Cambridge MA 02138; GaryKing.org, King@Harvard.edu, (617) 5007570.
    ${ }^{\text {§ }}$ Assistant Professor, University of Rochester, Harkness Hall, 333 Hutchinson Road, Rochester, NY 14627; scholar.harvard.edu/mkomisarchik; mkomisarchik@fas.harvard.edu, (720) 220-9328.

[^22]:    ${ }^{1}$ Claims about most other types of unfairness in redistricting all also seem to depend on a legal finding of noncompactness (Davis v. Bandemer, 478 U.S. 165; Justice Powell in Vieth v. Jubilerer, 541 U.S. 267 (2004) 176-177; Kirkpatrick v. Preisler, supra, at 394 U. S. 526, 538).
    ${ }^{2}$ The empirical claim sometimes implied in the law, that compactness requirements constrain racial or partisan gerrymandering, is the subject of active research program (Altman and McDonald, 2012; Barabas and Jerit, 2004; Chen and Rodden, 2013), and the role of compactness in ensuring other important normative virtues - such as better knowledge, communication, and trust between representatives and citizens - is also contested (Cain, 1984; Pildes and Niemi, 1993). But regardless of the outcome of these important debates, the degree of compactness of legislative districts will always have an essential role in defining the nature of representation and electoral competition in modern democracies, and an accurate measurement is essential to each debate.

[^23]:    ${ }^{3}$ Some states have passed laws highlighting certain features of compactness that may help with intuition but neither precision nor application. For example, Virginia Senate Joint Resolution 224 (1/14/2015, Article II, Section 6(5)) reads "Each legislative and congressional district shall be composed of compact territory. Districts shall not be oddly shaped or have irregular or contorted boundaries, unless justified because the district adheres to political subdivision lines. Fingers or tendrils extending from a district core shall be avoided, as shall thin and elongated districts and districts with multiple core populations connected by thin strips of land or water...." Iowa (Iowa Code, Title II §42.4) and Michigan (Congressional Redistricting Act 221 of 1999, Redistricting plan guidelines) mention some precise measures but not how to use this information.

    4"Indeed," writes Justice Souter, dissenting in Vieth v. Jubelirer, "although compactness is at first blush the least likely of these [traditional redistricting] principles to yield precision, it can be measured quantitatively in terms of dispersion, perimeter, and population ratios, and the development of standards would thus be possible."

[^24]:    ${ }^{5}$ Aspects of the overall methodology we develop here can also be applied to some other redistricting criteria, when additional data are available (or to concepts unrelated to redistricting that you only know when you see). These may include other characteristics of districts such as size; population equality across districts; where people live within a district (Fryer Jr and Holden, 2011); whether the district divides communities of interest or local political subdivisions; whether incumbents are paired or grouped in the same district and so have to run against each other to keep their jobs; what types of people are included in or excluded from a district; and, as a result, partisan fairness, electoral responsiveness (Gelman and King, 1994; Grofman and King, 2007), and racial fairness (King, Bruce, and Gelman, 1996). Redistricting also influences more personalistic factors common in real redistricting cases, such as whether a specific district includes features like a military base (which can influence a candidate's policy preferences) or a prison (which counts under "equal population" requirements but not votes), or even whether a candidate's parents homes or children's schools are drawn out of his or her district.

[^25]:    ${ }^{6}$ This pattern may be related to the "horizontal-vertical illusion" discovered in psychology (Prinzmetal and Gettleman, 1993).
    ${ }^{7}$ We note as well that, since all modern political boundaries are drawn with respect to cardinal directions, those directions are necessarily considered in examining districts.

[^26]:    ${ }^{8}$ We also experimented with having two coders participate together in ranking each set of districts, on the theory that the social connections would make the task even more engaging. Our results support this theory, in that respondents spent about $30 \%$ more time together completing the task, but this engagement was unnecessary since it did not increase inter- or intracoder reliability.

[^27]:    ${ }^{9}$ We define alphabetical order according to a specific naming convention. All districts receive an identifier which includes state, district set (upper chamber, lower chamber or Congress), district number, and year. For example, Alaska's first congressional district from 2010 is 01_CD_001_2010.

[^28]:    ${ }^{10}$ All districts are visualized at maximally high resolution to ensure that no features such as coastline are lost.

